Technological Diversification*

Miklós Koren

Federal Reserve Bank of New York, International Research Function

Silvana Tenreyro

Department of Economics, London School of Economics

Abstract

Why is GDP so much more volatile in poor countries than in rich ones? To answer this question, we propose a theory of technological diversification. Production makes use of different input varieties, which are subject to imperfectly correlated shocks. Technological progress takes the form of an increase in the number of varieties, raising average productivity. In addition, the expansion in the number of varieties in our model provides diversification benefits against variety-specific shocks and it hence lowers the volatility of output growth. Technological complexity evolves endogenously in response to profit incentives. The decline in volatility thus arises as a robust by-product of firms' incentives to increase profits and is hence an inexorable feature of the development process. We discuss the predictions of the model in light of the empirical evidence.

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1 Introduction

Economies at early stages of the development process are often shaken by abrupt changes in growth rates. In his influential paper, Lucas (1988) brings attention to this fact, noting that "within the advanced countries, growth rates tend to be very stable over long periods of time," whereas within poor countries "there are many examples of sudden, large changes in growth rates, both up and down."

Motivated by this empirical observation, this paper proposes an endogenous growth model of technological diversification. The key idea of the model is that firms using a large variety of inputs can mitigate the impact of shocks affecting the productivity of individual inputs. This takes place through two channels. First, with a larger variety of inputs, each individual input matters less in production, and productivity becomes less volatile by the law of large numbers. Second, whenever a shock hits a particular input, firms can adjust the use of the other inputs to partially offset the shock. Both channels make the productivity of firms using more sophisticated technologies less volatile.

The idea can be illustrated with an example from agriculture: Growing wheat with only land and labor as inputs renders the yield vulnerable to idiosyncratic shocks (for example, weather shocks such as a severe drought). In contrast, using land and labor together with artificial irrigation, fertilizers, pesticides, etc., can make wheat-growing not only more productive on average but also less risky, because farmers have more options to react to external shocks. Figure 1 provides a graphical illustration of this example. It displays the volatility of wheat yield (calculated as the standard deviation of percentage deviations from the country's average yield) of the 20 biggest wheat producers against their level of GDP per capita.[1] As shown, yield volatility falls sharply with development.[2]

A second, more topical example, can be drawn from the energy sector. According to The Economist, the recent increase in oil prices has led to a growing move towards ethanol and biofuels produced from canola and soya beans and thus less exposed to fluctuations in production and political turmoil.[3]

Our paper builds on the seminal contributions by Romer (1990) and Grossman and Helpman (1991) and characterizes technological progress as an expansion in the number

[1]Note that agricultural technology varies substantially with development. For example, of the top 20 wheat producers, India uses 2.3 tractors per 1,000 acres of arable land; this number is 128.8 for Germany. Fertilizer use also varies hugely. India uses 21.9 tons of nitrogenous fertilizers per acre; Germany uses 183.8 tons. We take the level of development as an overall indicator of agricultural sophistication.

[2]This remains true if we control for differences in climate across countries, including the volatility of rainfall and temperature.

[3]The Economist, 05/06/06, page 52, "Alternative Energy: Canola and Soya to the Rescue."

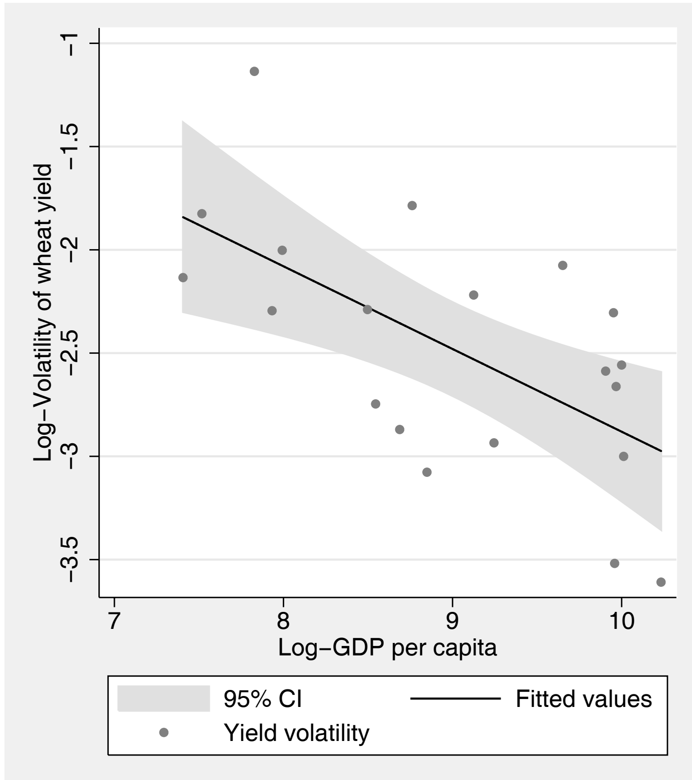

Figure 1: Wheat Yield Volatility and Development

of input varieties.[4] As is standard in these endogenous growth models, the number of varieties evolves in response to profit incentives, and increases in the number of varieties raise the level of average productivity. In addition, the expansion in the number of varieties in our model provides diversification benefits against variety-specific shocks and it hence lowers the volatility of output. In other words, the decrease in volatility in the model arises as a powerful by-product of firms' incentives to increase profits.

Our model also relates to a rich theoretical literature studying the link between volatility, diversification, and development, including Greenwood and Jovanovic (1990), Saint-Paul (1992), and Acemoglu and Zilibotti (1997). All these papers emphasize the role of *financial* rather than *technological* diversification as the mechanism through which volatility declines with development. The economics behind these models is in essence very different from ours. In particular, these models feature an inherent trade-off between productivity and risk at the firm (or agent) decision level: firms can choose between low-return, low-productivity activities and high-return, high pro-

[4]See also Barro and Sala-i-Martin (1995) for a comprehensive formalization and discussion of expanding-variety models.

ductivity ones. In that context, higher opportunities of financial diversification lead to more specialization in riskier but more productive activities.[5] Risk aversion is the crucial element generating the decline in volatility with the level of development. In the absence of risk aversion in these models, the economy would be specialized in the most productive activity and it would hence be extremely volatile. In contrast, the expanding-variety model of technological diversification we propose posits no trade-off between productivity and risk at the decision level; adopting a new variety increases productivity and decreases volatility. The incentive of firms to increase the number of varieties and hence lower volatility stems from the desire to increase profits (and hence productivity), as in Romer (1990) and Grossman and Helpman (1991), and does not hinge on the degree of risk aversion nor the financial infrastructure of the country. Technological diversification and the consequent decline in volatility are therefore robust outcomes accompaning the development of new varieties.

We view both margins of diversification for the firm, financial and technological, as complementary and empirically plausible. The purpose of this paper is to highlight the second margin.[6] The lack of trade-off between productivity and volatility in the model is motivated by the empirical patterns of sectoral specialization: As documented in Koren and Tenreyro (2007), countries at early stages of development tend to specialize in low-productivity, high-volatility activities, whereas the opposite pattern is observed at later stages. The development process is characterized by a move towards both more productive and safer sectors.

In departing from the portfolio-view models, our paper is closer to Kraay and Ventura (2001). This paper predicts that rich countries have a comparative advantage in less-volatile sectors. Specifically, high-skill-intensive sectors, which are prevalent in developed countries, face less-elastic product demand. Markups can then serve as a buffer against productivity shocks, reducing the volatility of high-skill sectors. The term-of-trade effect can then partly offsets the original shock. In low-skill sectors, instead, which tend to produce homogeneous products, no such "terms-of-trade insurance" takes place. The model hence predicts a negative relationship between productivity shocks and im-

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[5]In Acemoglu and Zilibotti (1997), for example, firm managers can issue financial securities and sell them to other agents in the stock market. In Greenwood and Jovanovic (1990), intermediaries can pool the risks of individual firms and offer a lower-risk, higher productivity asset. Financial development relaxes the constraints imposed by costly intermediation and thus leads to higher investment in riskier assets together with better opportunities for financial diversification.

[6]Technological diversification is also complementary to other other finance-related mechanisms emphasized in the literature. In particular, shocks can be amplified by introducing financial frictions, a task we do not undertake in the interest of clarity and simplicity. For models with financial frictions, see, among others, Bernanke and Gertler (1990), Kiyotaki and Moore (1997), Aghion, Angeletos, Banerjee and Manova (2004).

provement in terms-of-trade (particularly negative for developed countries). In the data, however, the relationship between fluctuations in labor productivity and terms of trade is somewhat positive, and there is no significant difference between rich and poor countries in terms of this relationship.[7]

The model naturally predicts a decline in volatility with the level of development. It is also consistent with two other robust facts about volatility: (i) The decline in volatility is more rapid in fast-growing countries. (ii) More productive and larger firms are less volatile. We present evidence on these predictions in the empirical section.

2 A Model of Technological Diversification

2.1 Production technology

This section introduces a production process that features technological diversification. Output Y is produced by combining a variety of inputs (or technologies) in a constant-elasticity-of-substitution (CES) production function,

$$Y = \left[\sum_{i=1}^{n} (\chi_i L_i)^{1-1/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}, \tag{1}$$

where L_i denotes the number of workers allocated to the operation of technology i, χ_i is the productivity of this variety, n denotes the number of varieties used by the producer, and $\varepsilon \in (1, \infty)$ is the elasticity of substitution across varieties.[8]

All technologies are symmetric *ex ante* (before the realization of productivity shocks) so that $L_i = L/n$ and $\chi_i = 1$ (by normalization) for all i, with L denoting the total number of employees working at the firm. We can then rewrite (1) as

$$Y = n^{1/(\varepsilon-1)} L. \tag{2}$$

Labor productivity (Y/L) is increasing in the number of varieties, since the varieties are imperfect substitutes ($\varepsilon < \infty$). This is the usual "love of variety" effect of many

[7]Of course it is possible that other factors are at play, blurring the predicted relationship and hence further empirical work should be done; at this point, nonetheless, we can say that the extent of countercyclicality in the terms of trade is not the *prima facie* mechanism behind the negative relationship between development and volatility.

[8]As usual in endogenous growth models, we assume that $\varepsilon > 1$, that is, technologies are gross substitutes. In an appendix available at request, we prove that the diversification result holds even with Leontief technologies (as in Kremer (1993)'s O-ring technology) as long as the shocks are not terminal. This is because more varieties allow the firm to adjust the use of more inputs, therefore providing more margins of adjustment, even if the inputs themselves are complements. Also note that introducing additional (scarce) factors of production would not change our qualitative results, it would just make the returns to variety more decreasing.

endogenous growth models (Romer 1990, Grossman and Helpman 1991). The effect is stronger the lower is ε, that is, the less substitutable varieties are. Intuitively, if varieties are highly substitutable, any additional variety is less needed. To rule out explosive growth, we assume $\varepsilon \geq 2$.[9]

Now suppose that variety-specific productivities are random, independently and identically distributed with mean $\mathrm{E}(\chi_i) = 1$ and variance $\mathrm{Var}(\chi_i) = \sigma^2$. We approximate the variance of output, a nonlinear function of productivity shocks, by linearizing (1) around the mean of each shock:

$$\hat{Y} = \sum_{i=1}^{n} \frac{\mathrm{MP}_i L_i}{Y} \left(\hat{\chi}_i + \hat{L}_i \right) = \frac{1}{n} \sum_{i=1}^{n} \hat{\chi}_i, \tag{3}$$

where $\hat{x} \equiv (x - \mathrm{E}\, x)/\, \mathrm{E}\, x$ denotes the infinitesimal deviation of variable x from its mean in proportional terms and MP_i denotes the marginal product of technology i. The last equality follows from Euler's theorem, the fact that varieties are *ex ante* symmetric, and full employment, which implies $\sum \hat{L}_i = \hat{L} = 0$.[10]

The proportional variance of output shocks is then

$$\mathrm{Var}\, \hat{Y} = \frac{\sigma^2}{n}. \tag{4}$$

The variance is declining in n, the number of technologies. This is a simple application of the law of large numbers: the variance of the average of n independent random variables is proportional to $1/n$.

2.2 The dynamics of technological diversification

What determines the level of technological complexity in the long run? In this section we endogenize the decision to invest in new varieties. Much as in models of endogenous growth, firm owners will be attracted by greater profit opportunities.

To spell out the dynamics of the model, we specify the stochastic properties of the productivity process, χ_i as follows. Time is continuous. Varieties have a constant productivity (normalized to 1) during their random lifetime, after which they irreversibly cease to contribute to production. The arrival of failures follows a Poisson process with arrival rate γ. In other words, the hazard rate of a failure is independent of how long the technology has been in use. Failures are independent across varieties.[11]

[9] Otherwise the love of variety effect would be so powerful that the aggregate return to varieties would become increasing. Higher levels of development would counterfactually imply increasing rates of return on capital inconsistent with observed development patterns (see Caselli and Feyrer (2006)).

[10] Since all firms are identical, full employment in the aggregate implies that there will be no changes in the level of employment at the firm level.

[11] We take the extreme assumption of independence for expositional clarity, but our argument goes through as long as failures are imperfectly correlated. The assumption that random failures turn the

Let $\chi_i(t)$ denote the productivity of technology i at time t and T_i the (random) lifetime of this technology. Productivity is 1 until time T_i, after which it falls to 0. Because failure arrives with a Poisson process, the lifetime follows an exponential distribution with parameter γ (the expected lifetime is hence $1/\gamma$). The probability that $T_i \leq t$ is thus

$$\Pr(T_i \leq t) = 1 - e^{-\gamma t}.$$

Clearly, the distribution of $\chi_i(t)$ is given by

$$\chi_i(t) = \begin{cases} 1 & \text{with prob. } e^{-\gamma t}, \\ 0 & \text{with prob. } 1 - e^{-\gamma t}. \end{cases}$$

To illustrate how shocks to technology affect the dynamics of a firm, let us follow a firm over time. Firms hire workers in competitive labor markets; at time t they face a wage rate $w(t)$ (taken as given by individual firms). The only state variable for the firm is the number of technologies that are currently in use, which takes value $n(t)$ at time t.[12] The marginal cost is given by:

$$\left[\sum_{i=1}^{n(t)} w(t)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} = w(t)n(t)^{1/(1-\varepsilon)}.$$

Firms using more varieties have lower marginal costs.

Suppose the firm faces a downward-sloping demand curve. In particular, assume that its demand is iso-elastic with elasticity η: $y(t) = Y(t)p(t)^{-\eta}$, where $y(t)$ is the firm's output, $Y(t)$ is aggregate output (taken as the numeraire), and $p(t)$ is the price charged by the firm. Profit maximization hence implies that the firm charges a constant $\eta/(\eta - 1)$ markup over its marginal cost:

$$p(t) = \frac{\eta}{\eta - 1} w(t)n(t)^{1/(1-\varepsilon)},$$

and its revenues are given by

$$p(t)y(t) = Y(t)p(t)^{1-\eta} = Y(t)\left[\frac{\eta w(t)}{\eta - 1}\right]^{1-\eta} n(t)^{(1-\eta)/(1-\varepsilon)}. \tag{5}$$

Profits are a constant $1/\eta$ fraction of revenues,

$$\pi(t) = p(t)y(t)/\eta = \frac{1}{\eta}Y(t)\left[\frac{\eta w(t)}{\eta - 1}\right]^{1-\eta} n(t)^{(1-\eta)/(1-\varepsilon)} = A(t)n(t)^{(1-\eta)/(1-\varepsilon)},$$

input completely useless makes the model more tractable; however, technological diversification would still take place with less terminal shocks.

[12] The symmetry assumptions above ensure that firms only care about the *number* of technologies but not about *which* technologies they use.

where $A(t) \equiv \frac{1}{\eta} Y(t) \left[\frac{\eta w(t)}{\eta - 1}\right]^{1-\eta}$. Since an individual firm takes $Y(t)$ and $w(t)$ as given, $A(t)$ is also given from a firm's perspective.

For analytical convenience, we assume the elasticity of demand (η) to be the same as the elasticity of substitution between varieties (ε). This assumption ensures that profits are linear in the number of varieties. The assumption is satisfied naturally if varieties represent different brands valued by the consumer, in which case the elasticity of demand and the elasticity of substitution are equal.

2.2.1 Technology adoption

Adopting new varieties is a costly and risky activity. The firm has to devote scarce resources to adoption efforts, and these efforts pay off only after a random waiting time. More intensive adoption efforts result in a shorter expected waiting time for the next variety. Adoption costs are independent of how much of the variety the firm wishes to use later. In this sense, adoption costs can be construed as fixed costs and the model features increasing returns to scale.[13,14]

The adoption of a new variety requires both a stock of knowledge (embedded in current technologies, $n(t)$) and a flow of investment. If the firm spends I units of the final good to adopt a new variety, the adoption will be successful with a Poisson arrival rate $\Lambda = f(I, n)$, where $f(.,.)$ is a standard neoclassical production function subject to constant returns to scale and satisfying the Inada conditions.[15] Let $\lambda = \Lambda / n$ denote the adoption *intensity*. By the CRS property of f, the flow cost of this adoption intensity is

$$I = g(\lambda) n,$$

where $g(.)$ is the inverse of $f(., 1)$, an increasing, convex function with $g(0) = g'(0) = 0$, $\lim_{x \to \infty} g'(x) = \infty$.

As mentioned, technological diversification in this model is not driven by risk aversion. To stress this point, we next characterize the optimal rate of technology adoption in the case of risk neutral agents. In Appendix A we characterize adoption under complete financial autarky and risk averse investors. We do this to highlight that there is technological diversification in both cases and that the incentive to diversify does not

[13] Adoption costs can be also thought as the cost of research and development of new varieties. For developing countries, however, referring to adoption (or imitation) costs seems more appropriate.

[14] A similar assumption is made by Acemoglu and Zilibotti (1997) who work with minimum scale requirements at the industry level.

[15] This formulation follows Klette and Kortum (2004). The random, "memoryless" adoption process ensures that we do not have to track past R&D investment flows of the firm. This is a standard simplifying assumption in endogenous growth models.

hinge on the financial structure of the economy nor the degree of risk aversion (though they may magnify these incentives).

Risk neutral households maximize the present value of consumption, discounted at the rate ρ:

$$\mathcal{U} \equiv \int_{t=0}^{\infty} e^{-\rho t} C(t)\,\mathrm{d}t.$$

The Euler equation pins down the riskless rate in the economy at $r(t) = \rho$. Investors maximize the expected present value of profits, discounted with the discount rate ρ. To ensure non-negative growth and a finite value for the firm, we further assume that the cost of adoption satisfies $g(\gamma) + \rho g'(\gamma) \leq L/2$ and $g(\gamma + \rho) > L/2$.

Let $V_n(t)$ denote the expected present discounted value of profits for a firm with n varieties at time t.

$$V_n(0) = \mathrm{E}_0 \int_{t=0}^{\infty} e^{-\rho t}[\pi(t) - I(t)]\,\mathrm{d}t = \mathrm{E}_0 \int_{t=0}^{\infty} e^{-\rho t}\{A(t) - g[\lambda(t)]\}n(t)\,\mathrm{d}t, \tag{6}$$

and the stochastic dynamics of $n(t)$ is described as follows. In each infinitesimal time period of length h, one of the technologies fails with probability $\gamma n h$ (omitting higher order terms), decreasing n by 1, or the firm becomes successful in adopting a new technology (with probability $\lambda n h$), increasing n by 1.

The Bellman equation describing the decision problem and the value of the firm is

$$\begin{aligned} \rho V_n(t) = \max_{\lambda} \{ & A(t)n - g(\lambda)n \\ & + \lambda n\,[V_{n+1}(t) - V_n(t)] \\ & + \gamma n\,[V_{n-1}(t) - V_n(t)] \\ & + \lim_{h \to 0} \mathrm{E}_t\,[V_n(t+h) - V_n(t)]/h \}. \end{aligned} \tag{7}$$

The opportunity cost of the value of the firm ($\rho V_n(t)$) equals the sum of (i) flow profits net of adoption costs ($A(t)n - g(\lambda)n$), (ii) capital gain from successful adoption of a new technology (which occurs with hazard rate λn), (iii) capital loss if any of the n variety fails (each of which has a hazard rate γ), and (iv) exogenous capital gains (due to changes in the aggregate environment affecting profitability).

Proposition 1. The optimal adoption rate is $\Lambda = \lambda(t)n$, so the adoption *intensity* is independent of n. The value of the firm is of the form $V_n(t) = v(t)n$, where $v(t)$ and $\lambda(t)$ are jointly determined by

$$g'[\lambda(t)] = v(t), \tag{8}$$

$$\left[\rho + \gamma - \lambda(t) - \frac{\mathrm{E}_t(\mathrm{d}v/v)}{\mathrm{d}t}\right] v(t) = A(t) - g[\lambda(t)]. \tag{9}$$

Adoption intensity, $\lambda(t)$, is positive and unique. For a given growth path, $\mathrm{E}_t(\mathrm{d}v/v)/dt$, it is increasing in profitability, $A(t)$; decreasing in the discount rate, ρ; decreasing in the probability of a technology shock, γ.

The term $\mathrm{E}_t(\mathrm{d}v/v)/\,\mathrm{d}t$ captures the expected capital loss due to the fact the profits fall (at the least, do not increase) over time, as we show in Section 2.3. The proof of this and all the remaining propositions are in the appendix.

The linearity of the problem ensures that the firm's problem is scale independent. The intensity of adoption is constant, so firms grow at a constant expected rate, independently of n. We can now fully characterize the dynamics of a firm.

Proposition 2. Conditional on the dynamics of aggregate variables, $Y(t)$ and $w(t)$, expected sales growth of the firm is $\lambda(t) - \gamma$, and the variance of sales growth is $[\lambda(t) + \gamma]/n(t)$.

Equation (5) shows that, conditional on aggregate variables, sales is a linear function of n, hence its growth rate equals the growth rate of n. The number of varieties is expected to grow with a rate that equals the speed of technology adoption minus the speed of technology failures, $\lambda - \gamma$. The variance of sales growth is driven by the two shocks the firm faces: the randomness of the adoption process and technology failures. Hence the variance of an individual variety is $\lambda + \gamma$. Total sales volatility then declines with n by the law of large numbers. (The Appendix gives a formal proof.)

The result indicates that bigger, more productive firms are less volatile,[16] which is in line with stylized fact number 3.

2.3 Aggregate dynamics

So far we have taken aggregate output $Y(t)$ and the wage rate $w(t)$ as given, independent of $n(t)$. In general equilibrium, however, more varieties lead to higher productivity, resulting in higher output and wages. These in turn affect firms' profitability and their path of adoption. We close the model by considering these linkages.

There is a unit mass of identical firms, indexed by i. Firm i has $n(i,t)$ varieties. The output of the final good is a CES aggregate of firm-level outputs,

$$Y(t) = \left[\int_{i=0}^{1} y(i,t)^{(\varepsilon-1)/\varepsilon}\,\mathrm{d}i\right]^{\varepsilon/(\varepsilon-1)} = N(t)^{1/(\varepsilon-1)}L, \tag{10}$$

where L denotes the fixed labor supply, $N(t) = \int_{i=0}^{1} n(i,t)\,\mathrm{d}i$ is the aggregate number of varieties and individual firm output comes from equation (2).

[16] As is generally the case in monopolistic competition models, n is an index of both productivity and size.

Aggregate productivity is increasing in the aggregate number of varieties, $N(t)$. In a symmetric equilibrium, all firms use the same number of varieties, $n(i,t) = N(t)$. We assume that both technology shocks and the random success of adoption hit all firms at the same time. In other words, the shocks are technology-specific, not firm-specific. This ensures that they have aggregate effects and there is aggregate volatility. So, for example, an increase in oil prices (which in the model materializes as a drop in productivity χ—more units of final output are needed to operate one unit of oil-dependent inputs) affects all firms in the same way since they are ex-ante identical.

The specification of technology shocks and the adoption process ensure that $N(t)$ is the single state variable for the economy. Output, wage, and the adoption rate are deterministic functions of N at any point in time. The dynamics of N is as follows. With hazard rate $\lambda(N)N$, adoption is successful and N jumps to $N+1$. With hazard rate γN, one of the varieties fails and N jumps to $N-1$.

We can then describe the volatility of the growth rate of Y as

$$\begin{aligned}\mathrm{Var}(\mathrm{d}Y/Y) &= \frac{\lambda(N)N\left[(N+1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}\right]^2 + \gamma N\left[(N-1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}\right]^2}{N^{2/(\varepsilon-1)}}\,\mathrm{d}t \\ &\approx \left(\frac{1}{\varepsilon-1}\right)^2 \frac{\lambda(N)+\gamma}{N}\,\mathrm{d}t, \end{aligned} \tag{11}$$

where the result uses the first-order approximation of $(N+1)^{1/(\varepsilon-1)} - N^{1/(\varepsilon-1)}$ as $N^{(2-\varepsilon)/(\varepsilon-1)}/(\varepsilon-1)$, and is exact for $\varepsilon = 2$.

Aggregate volatility depends on the randomness of adoption and technology failures. Because $\lambda(N)$ is non-increasing in N, volatility is decreasing in N, which we can think of as an index of economic development (equation (10)). Proposition 3 summarizes this key result, which is consistent with empirical patterns (see Section 3).

Proposition 3. The volatility of GDP growth rates declines with the level of GDP per capita.

To derive equilibrium wages, note that each firm has a constant profit margin $(1/\varepsilon)$. The total wage bill is a fraction $1-1/\varepsilon$ of total output, which pins down the wage rate at

$$w(N) = \left(1 - \frac{1}{\varepsilon}\right) N^{1/(\varepsilon-1)}. \tag{12}$$

Equations (10) and (12) together imply that the demand shifter of the firm is

$$A(N) = \frac{1}{\varepsilon} N^{(2-\varepsilon)/(\varepsilon-1)} L. \tag{13}$$

Individual firm profits decrease with aggregate productivity (or remain unchanged when $\varepsilon = 2$). There are two opposing forces at play. On the one hand, since varieties are substitutes, higher productivity of competitors implies lower demand for a particular firm's product. On the other hand, since varieties are *imperfect* substitutes, there is a demand externality: more aggregate varieties raise income and hence demand for every firm's product. As long as the elasticity of substitution is not too low, the first effect dominates. If $\varepsilon = 2$, the two effects exactly cancel out, and we obtain a balanced growth path, as summarized in the following proposition.

Proposition 4. If the elasticity of substitution is $\varepsilon = 2$, the economy grows at a constant *mean* growth rate x, $\mathrm{E}(\mathrm{d}Y/Y) = x\,\mathrm{d}t$. If the elasticity of substitution is $\varepsilon > 2$, the economy has a stochastic steady state, in which N (and hence Y) has a steady state distribution.

In the balanced growth case, the expected growth rate x is implicitly defined by

$$[\rho - x]\, g'(\gamma + x) = L/2 - g(\gamma + x), \tag{14}$$

and our assumptions ensure that $x \in [0, \rho)$. The growth rate is increasing in L; decreasing in the discount rate, ρ; decreasing in the probability of a technology shock, γ; and decreasing with an upward shift in $g(.)$ (costlier adoption).

If $\varepsilon > 2$, we can characterize the steady state distribution of N with its mode, N^*, which is implicitly defined by $\lambda(N^*) = \gamma$.[17] Substituting this in equations (8), (9), and (13),

$$N^* = \left[\frac{\rho g'(\gamma) + g(\gamma)}{L/\varepsilon}\right]^{\frac{\varepsilon-1}{2-\varepsilon}}. \tag{15}$$

The mode N^* is increasing in L; decreasing in ρ; decreasing in γ; and decreasing with an upward shift of $g(.)$.

If N is below its mode, it is *expected* to increase with the rate $\mathrm{E}(\mathrm{d}N/N) = [\lambda(N) - \gamma]\,\mathrm{d}t$. Similarly, if N is above its mode, it is expected to decrease. However, because of the stochastic shocks, it also has a positive probability of moving *away* from N^*.

3 Volatility and Development: Empirics and Discussion

The model developed in the previous sections is consistent with three robust empirical regularities. We discuss these regularities in conjunction with the predictions of the model.

[17] Strictly speaking, because N^* can only be an integer, it is defined as the lowest N for which $\lambda(N) \leq \gamma$.

Fact 1. GDP volatility declines with development, both in the cross section, and for a given country over time.

The decline in volatility with the level of development is one of the stylized facts in the literature and the main motivation of this paper. There are large cross-country differences in volatility. The standard deviation of annual GDP growth during the period 1970 through 2000 ranges from 1.4 percent to 21.8 percent (or a factor of 15) across 167 countries. The cross-country variation in volatility is highly correlated with the cross-country variation in the level of development, gauged by real GDP per capita. This is illustrated in the left hand-side panel of Figure 2, which plots the (log) level of volatility, measured as the standard deviation of growth rates over non-overlapping decades from 1960 through 2000, against the average (log) level of real GDP per capita over the decade. The graph also shows the linear regression line together with the 95-percent confidence-band intervals. In the model, the cross-sectional decline in volatility results naturally as countries with a higher degree of techological sophistication enjoy higher productivity and lower volatility levels.

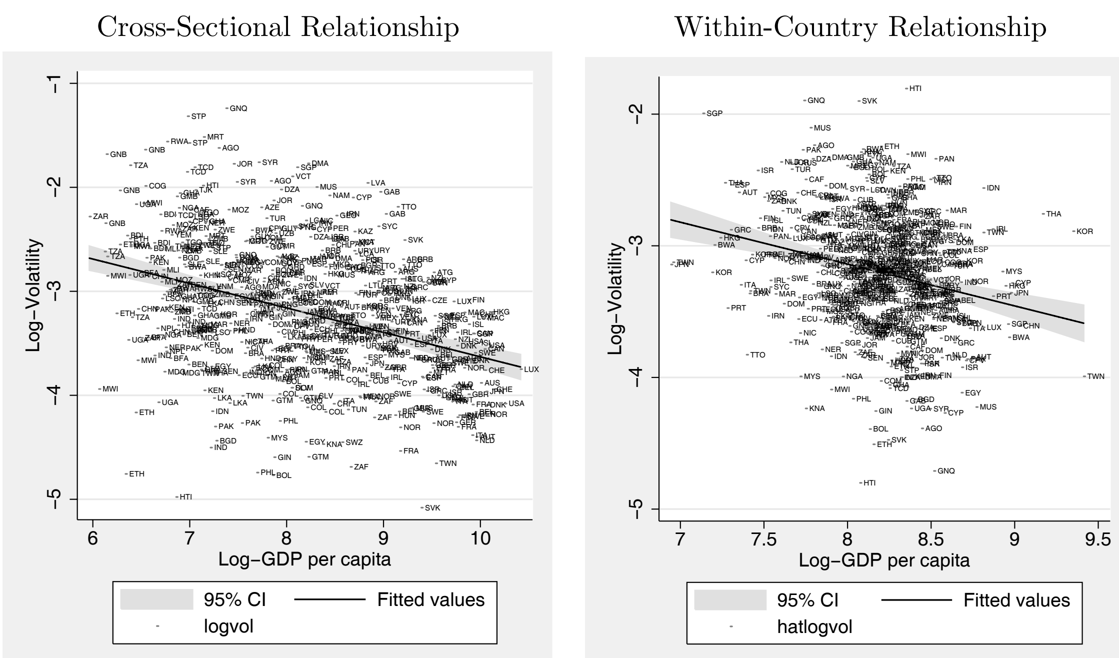

Figure 2: Volatility and Development

The model also predicts that, for a given country over time, volatility declines with development. This is illustrated in the second panel of Figure 2, which plots the same variables after controlling for country-specific effects. In other words, keeping country characteristics (e.g., geography, institutions) constant, growth and changes in volatility are negatively correlated. This negative correlation holds at different levels of financial development, as illustrated in Figure 3. In this Figure, we split the level of financial development, measured as is standard, by the (log) ratio of private credit to GDP, into four quartiles. Similar results obtain by splitting financial development in narrower quantiles.[18]

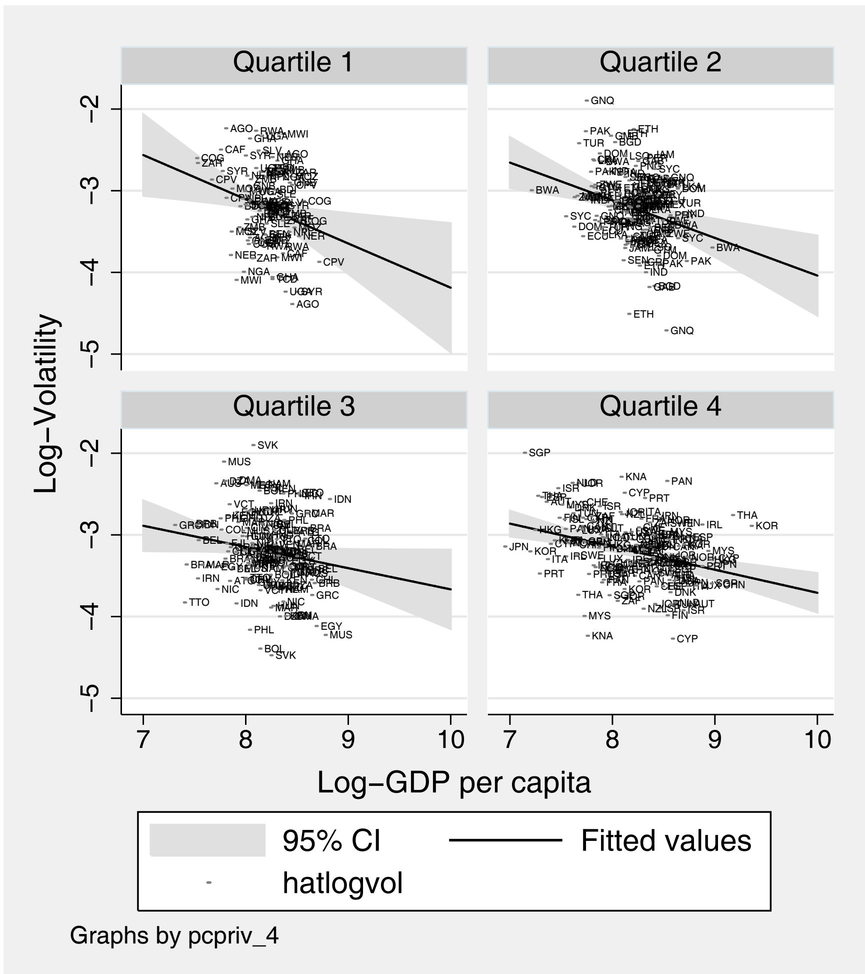

Figure 3: Volatility and Development by Financial Development Quartile

The graphs hence indicate that the decline of volatility with development is not sensitive to the level of financial development of the country. Even countries with very limited financial infrastructure experience a decline in volatility when they grow.

[18]In related work, Ramey and Ramey (1995) study the link between volatility and growth. Here, as suggested by the model, we study the links between volatility and *productivity* or between *changes* in volatility and growth.

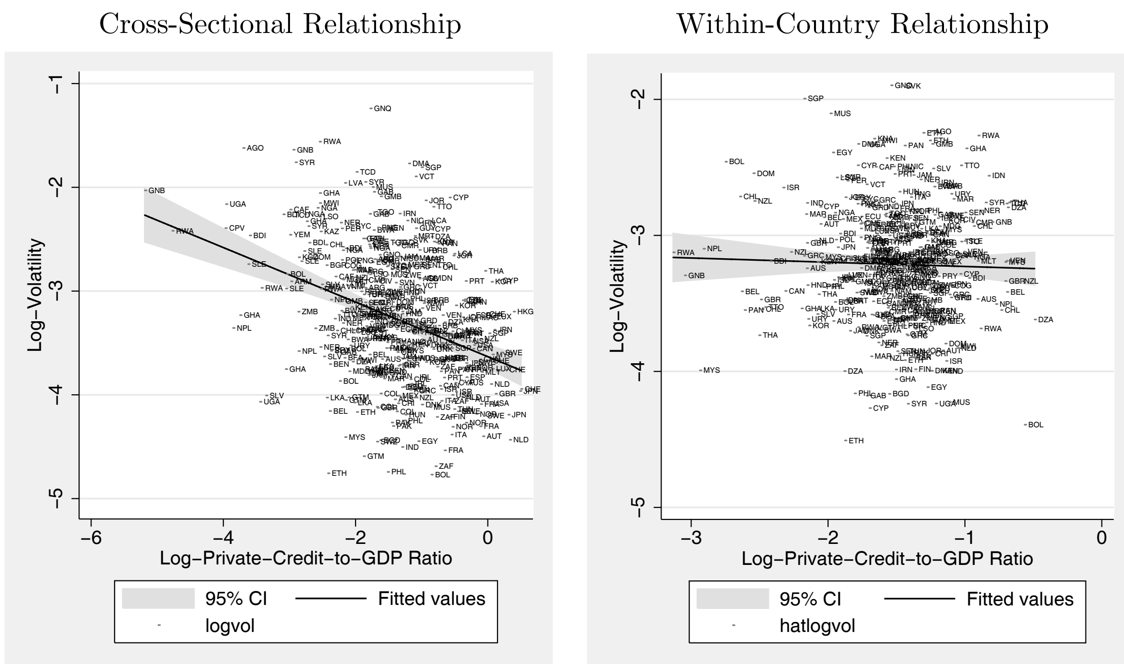

Figure 4: Volatility and Financial Development

For completeness, we show in Figure 4 the cross-sectional and within-country relationship between volatility and financial development. As expected, we observe a strong negative correlation between volatility and financial development in the cross-section. However, the correlation vanishes once we control for country-specific effects. This result is interesting as it suggests that the decline in volatility for a given country over time cannot be explained in a statistical sense by higher levels of financial development.

Table 1: Volatility, Development, and Finance

	Dependent Variable: Standard Deviation of Growth Rates					
GDP per capita (constant PPP $)	-0.2319***	-0.3008***			-0.1689***	-0.3153***
	[0.0318]	[0.0622]			[0.0454]	[0.1169]
Private Credit / GDP			-0.1468***	-0.0198	-0.0711*	0.0124
			[0.0542]	[0.0489]	[0.0380]	[0.0539]
Country Fixed Effects	No	Yes	No	Yes	No	Yes
Observations	585	585	403	403	403	403
R-squared	0.13	0.56	0.09	0.60	0.13	0.62

Note: All variables are in logs. The equations use the 10-year standard deviation of annual growth rates from 1960 to 2000. The regressors are computed at their mean values over the decade. Clustered standard errors in brackets. * significant at 10%; ** significant at 5%; *** significant at 1%.

We summarize these correlations in Table 1. The first two columns shows the coefficients from a regression of (log) volatility on real GDP per capita, excluding and including fixed effects. The coefficients are statistically significant at the 1 percent level. The third and fourth columns show the corresponding results when volatility is regressed on the (log) ratio of private credit to GDP. As anticipated earlier in the graphs, the cross sectional relationship is strongly negative. However, once fixed effects are included, the estimated elasticity is both statistically and economically insignificant. Finally the last two columns show the regression results when both variables are included in the regression. Volatility is strongly (and negatively) associated with the level of per capita GDP, while there is little or no (partial) correlation with the level of financial development.

In the model, the high volatility at early stages of development results from the relatively low number of varieties used in the production process. Various empirical studies document the low or delayed adoption of varieties in developing countries. For example, Caselli and Coleman (2000) find that the adoption of computers depends crucially on the level of development of the country. Caselli and Wilson (2004) show that this result extends to a broader set of high-technology equipment (where the extent of technology embodied in capital equipment is measured as the R&D content). Comin and Hobijn (2004) provide additional support for this observation: They document how specific technological innovations have spread across countries. The authors show that most innovations originated in developed countries and spread gradually to less-developed countries. This implies that at any point in time poor countries use fewer varieties than rich ones.

Fact 2. Manufacturing is both more productive and less volatile than agriculture.

To the extent that manufacturing uses more complex production technologies than agriculture, the model predicts that manufacturing sectors should be both more productive and less volatile than agriculture. This is indeed consistent with a strong regularity in the data: On average, volatility of value-added per worker in agriculture is around 50 percent higher than that in manufacturing. At the same time, value added per worker is around twice as high in manufacturing than in agriculture. These figures are computed from the OECD-STAN and summarize the figures reported in Table 2. The table shows the average of labor productivity in manufacturing relative to labor productivity in agriculture from 1970 through 2003 and the corresponding ratio of volatilities over the same period. In all countries, manufacturing is significantly more productive, as predicted by the model. Moreover, manufacturing is also less volatile, with the only exception of Italy, where volatility is slightly higher in manufacturing.

Fact 3. More productive firms are less volatile.

Table 2: Ratio of Manufacturing-to-Agriculture Productivity and Volatility

Country	Relative Productivity in Manufacturing	Relative Volatility in Manufacturing
Australia	1.41	0.20
Austria	6.82	0.42
Belgium	2.08	0.45
Canada	1.72	0.47
Denmark	1.19	0.36
Finland	2.17	0.84
France	1.66	0.31
Germany	2.34	0.43
Greece	1.58	0.97
Italy	1.77	1.01
Japan	4.28	0.50
Korea	2.51	0.45
Luxembourg	1.78	0.35
Netherlands	1.37	0.37
Norway	1.54	0.73
Poland	4.23	0.36
Portugal	2.18	0.43
Spain	1.84	0.33
Sweden	1.46	0.65
United Kingdom	1.52	0.42
United States	2.24	0.23

Note: Column 2 shows the ratio of average labor productivity in manufacturing over labor productivity in agriculture from 1970 to 2003. Column 3 shows the corresponding ratio for standard deviation of labor productivity growth during the period.

A crucial point in the model is that, at the firm level, average productivity and volatility are negatively correlated. Table 3 shows the partial correlations between the volatility of sales growth, average size (employment) and productivity (sales per worker) for 9000 Compustat firms. Volatility is calculated for non-overlapping decades from 1950 through 2000. Both productivity and size are negatively correlated with firm-level volatility. This remains true if we include firm-fixed effects to look at the time-series variation only: firms becoming more productive also become more stable. This is consistent with our result presented in Proposition 2: Productivity (and employment) growth is associated with the adoption of new varieties, technological diversification, and hence lower volatility.

Table 3: Firm Productivity and Volatility

	Dependent Variable: Standard Deviation of Growth Rates			
Sales per worker	-0.115***		-0.137***	-0.136***
	[0.008]		[0.007]	[0.017]
Employment		-0.192***	-0.194***	-0.198***
		[0.002]	[0.002]	[0.009]
Firm Fixed Effects	No	No	No	Yes
Decade Fixed Effects	Yes	Yes	Yes	Yes
Observations	25408	25408	25408	25408
R-squared	0.06	0.24	0.26	0.26

Note: All variables are in logs. The equations use the 10-year standard deviation of annual sales growth rates from 1950 to 2000. The regressors are computed at their mean values over the decade. Clustered standard errors in brackets. * significant at 10%; ** significant at 5%; *** significant at 1%.

4 Conclusion

This paper proposes a model in which the production process makes use of different input varieties subject to imperfectly correlated shocks. As in other endogenous-growth models, technological progress takes place as an expansion in the number of input varieties, increasing productivity. The new insight in the model is that the expansion in varieties also leads to lower volatility of production via two channels. First, as each individual variety matters less and less in production, the contribution of idiosyncratic fluctuations to overall volatility declines. Second, each additional input provides a new adjustment margin in response to external shocks, making productivity less volatile.

In the model, the number of varieties evolves endogenously in response to profit incentives and the decrease in volatility comes out as a powerful by-product of firms' incentives to increase profits.

The extant literature on the link between volatility and development has emphasized the role of *financial* rather than *technological* diversification as the mechanism through which volatility declines with development. In this literature, risk aversion is the crucial element generating the decline in volatility with the level of development. In the absence of risk aversion, the economy would specialize in the most productive activity and it will be extremely volatile. In contrast, in the expanding-variety model of technological diversification, the incentive of firms to increase the number of varieties and hence lower volatility stems from the desire to increase productivity, and does not hinge on the degree of risk aversion or the financial infrastructure of the country. In practice, both margins of diversification for the firm, financial and technological, can be of course viewed as complementary; the purpose of this paper is to highlight the second margin.

The model yields empirical predictions concerning the relationships between volatility and productivity at the aggregate and firm level. We discuss these predictions in light of the empirical evidence.

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Appendix

A Technology Adoption under Complete Financial Autarky

In this Appendix we discuss technology adoption when risk pooling is not possible. Each firm is owned by a risk-averse individual, whose only income is the profit of the firm. Utility exhibits risk aversion with $u' > 0$, $u'' < 0$, $u(0) > -\infty$, $u'(0) < \infty$. These latter assumptions ensure the finiteness of the value of the firm even if there is a positive probability that the firm profits (and hence consumption) will eventually become zero.

The value of the firm is defined as lifetime expected utility,

$$V_n(t) \equiv \mathrm{E}_t \int_{s=t}^{\infty} e^{-\rho s} u\{A(t)n - g[\lambda(t)]n\}\, \mathrm{d}t. \tag{16}$$

The Bellman equation characterizing the firm's problem is

$$\begin{aligned} \rho V_n(t) = & \max_{\lambda} \{u[A(t)n - g(\lambda)n] \\ & + \lambda n \left[V_{n+1}(t) - V_n(t)\right] \\ & + \gamma n \left[V_{n-1}(t) - V_n(t)\right]\} \\ & + \lim_{h \to 0} \mathrm{E}\left[V_n(t+h) - V_n(t)\right]/h\}, \end{aligned} \tag{17}$$

$$\text{s.t.} A(t) - g(\lambda) \geq 0. \tag{18}$$

This is the same as (7) with the exceptions that (i) flow utility is a concave function of firm profits, and (ii) that we rule out borrowing so that adoption has to be financed from current profits.

Proposition 5. Optimal technology adoption intensity, $\lambda(n, t)$ is strictly positive for all $n > 0$ and t.

Proof. Because $g(0) = 0$, the non-negative profit constraint provides a *positive* upper bound on λ. If the constraint is binding, λ is positive. Otherwise we can use the first-order-condition for optimal adoption,

$$u'[A(t)n - g(\lambda)n]g'(\lambda) = V_{n+1}(t) - V_n(t). \tag{19}$$

The properties of u' and g' ensure that there will be a unique positive λ for each n as long as $V_{n+1} - V_n > 0$. This condition is easy to verify. It is obvious that $V_{n+1} \geq V_n$, because the firm can always throw away the additional variety and replicate its profits with n varieties. We can also show that it is strictly better off with more varieties.

The value of a firm with n_0 products is V_{n_0} defined by (16). Now calculate a lower bound for the expected discounted utility if the firm adds a variety. Suppose the firm does not change its adoption efforts but keeps them at $\lambda(n_0)$. Let us denote the value of this strategy by $\tilde{V}_{n+1}$. It is clear that $V_{n+1} \geq \tilde{V}_{n+1}$, because the firm cannot lose by adjusting its adoption intensity optimally.

The flow profits the additional variety generates while working is strictly higher than the previous profits:

$$\tilde{\pi}(t) - \pi(t) = \{A(t)[n(t) + 1] - g[\lambda(t)]\} - \{A(t)n(t) - g[\lambda(t)]\} = A(t) > 0,$$

which ensures $\tilde{u}(t) > u(t)$ for all $t \leq T_{n+1}$, because $u' > 0$ even if the consumer is risk averse. Because the new variety is expected to have a positive lifetime ($T_i > 0$ with probability 1), we have that $\tilde{V}_{n+1} > V_n$ and hence $V_{n+1} > V_n$. □

The proof relies on the property that new varieties lead to higher profits. This is why firms have an incentive for technological diversification even in the complete absence of financial markets. Of course, the *magnitudes* may vary with the degree of financial development and technology adoption may be faster or slower in financial developed economies. However, we demonstrated that financial deepening is not *required* for technological diversification to work.

The result that the adoption intensity is positive for *all* n depends on the functional form assumptions about the cost of adoption. In particular, the Inada conditions ensure that it is always optimal to devote some resource to adoption as long as the marginal benefit is positive. Of course, if the marginal cost of adoption is bounded away from zero, there is a range of positive but small marginal benefits for which adoption intensity will be zero. This does not alter the result that financial development is not a *necessary condition* for technological diversification.

B Proofs

Proof of Proposition 1. Since profits are linear in n, guess that the form of value function is $V_n(t) = v(t)n$. Equation (8) is then the first-order condition for optimal λ. Equation (9), in turn, results from substituting the guess function into the Bellman equation. From (8), λ is independent of n.

Let $\phi(t)$ denote expected capital gain per variety, $\mathrm{E}_t(\mathrm{d}v/v)/\mathrm{d}t$. Substituting (8) into (9),

$$[\rho + \gamma - \phi(t)]\, g'(\lambda) = A(t) - g(\lambda) + \lambda g'(\lambda)$$

Both sides are continuously differentiable, the LHS is 0 at $\lambda = 0$, the RHS is positive. At $\lambda = \gamma + \rho$, the LHS is lower than the RHS by Assumption **??**. In between, the LHS is growing faster than the RHS, so there is a unique $\lambda \in (0, \gamma + \rho)$.

This gives λ and hence $v = g'(\lambda)$ as an increasing function of both ϕ and A. This can be inverted to obtain $\phi = h(v, A)$, which is increasing in v and decreasing in A. Intuitively, holding the value of the firm constant, increasing profits today requires that we lower profits tomorrow, hence lower ϕ.

□

Proof of Proposition 2. The proof follows directly from the following lemma.

Lemma 1. Let x_t follow a discrete-state Markov process with Poisson jumps between states $\{E_1, E_2, ..., E_N\}$, with transition probabilities $\Pr(x_{t+h} = E_n | x_t = E_k) = \xi_{n,k} h + o(h)$. Then (i) the expected change in x_t is

$$\mathrm{E}(\mathrm{d}x_t | x_t = E_k) = \sum_{i=1}^{N} \xi_{i,k}(E_i - E_k)\, \mathrm{d}t,$$

and (ii) its instantaneous variance is

$$\mathrm{Var}(\mathrm{d}x_t | x_t = E_k) = \sum_{i=1}^{N} \xi_{i,k}(E_i - E_k)^2\, \mathrm{d}t.$$

(i) The first jump arrives with arrival rate $\sum_{i=1}^{N} \xi_{i,k}$. Conditional on a jump occurring, the probability of state n is $\xi_{n,k}/\sum \xi$, and the jump size is $(E_n - E_k)$ in this case. Taking expectation over all possible states, we get the result. (ii) Note that the instantaneous volatility equals the instantaneous second moment, $\mathrm{E}[(\mathrm{d}x_t)^2 | x_t = E_k]$, because $\mathrm{E}(\mathrm{d}x_t | x_t = E_k)^2$ is of order $O(\mathrm{d}t^2)$. Then applying (i) to jumps of size $(E_n - E_k)^2$, we obtain the result.

Substitute in $E_n = n$ and $\xi_{i,k} = \lambda k$ if $i = k + 1$, $\xi_{i,k} = \gamma k$ if $i = k - 1$, and $\xi_{i,k} = 0$ otherwise. Then $\mathrm{E}(\mathrm{d}n) = (\lambda - \gamma) n\, \mathrm{d}t$ and $\mathrm{Var}(\mathrm{d}n) = (\lambda + \gamma) n\, \mathrm{d}t$. Divide by n to obtain the result. □

Proof of Proposition 3. Equation (11) expresses the volatility of GDP growth rate as $[\lambda(N) + \gamma]/N$. We show that this is declining in N because $\lambda(N)$ is nonincreasing in N. Because N is positively related to GDP per capita, Y/L, this will complete the proof.

The value of the firm that has n varieties in an economy with N aggregate varieties is

$$V(N, n) = v(N)n = \mathrm{E} \int_{t=0}^{\infty} e^{-\rho t}[A(N_t) - g(\lambda_t)]n_t \, \mathrm{d}t.$$

Equation (13) shows that profitability per variety is non-increasing in N. An economy that starts from a lower $N' < N$ has greater or equal profitability not only today but in all future time periods and states of the world, $A'(t, h) \geq A(t, h)$ for all time t and history of shocks h. The present discounted value of the higher profits is higher even if the firm does not adjust its adoption policy and potentially even higher thereafter, $V(N', n) \geq V(N, n)$. This implies that $v(N') \geq v(N)$ for all $N' < N$, so that $v(N)$ is nonincreasing in N. From (8) and the strict convexity of g, innovation is a positive function of v, so $\lambda(N)$ is nonincreasing in N. □

Proof of Proposition 4. (i) With $\varepsilon = 2$, aggregate output is $Y = NL$, and profitability does not depend on N, $A = L/\varepsilon$. Then from (8) and (9), we have that $v(t) = v$ constant, which makes $\lambda(t) = \bar{\lambda}$ constant. Aggregate dynamics then looks the same as firm-level dynamics described in Proposition 2.

(ii) The dynamics of aggregate N is characterized as a birth and death process with birth rate $\xi_N = \lambda(N)N$ and death rate $\mu_N = \gamma N$. A birth and death process has a steady state distribution if the series

$$\sum_{k=1}^{\infty} \frac{\xi_1 \xi_2 \cdots \xi_k}{\mu_2 \mu_3 \cdots \mu_{k+1}}$$

converges. Substituting in ξ_k and μ_k, this condition becomes

$$\sum_{k=1}^{\infty} \gamma^{-k} \left[\lambda(1)\lambda(2) \cdots \lambda(k)\right] < \infty.$$

The proof of Proposition (3) shows that $\lambda(N)$ is a nonincreasing function of N. In fact, because profits tend to zero as N tends to infinity, we have $\lim_{N \to \infty} \lambda(N) = 0$. This implies that for a $\delta < 1$ there exists a $K < \infty$ such that $\lambda(N) < \delta\gamma$ for all $N > K$.

The product $\lambda(1)\lambda(2) \cdots \lambda(k)$ is then bounded from above by $\lambda(1)\lambda(2) \cdots \lambda(K)\delta^{k-K}\gamma^{k-K}$ for $k < K$. The series is bounded by

$$\lambda(1)\lambda(2) \cdots \lambda(K)\delta^{k-K}\gamma^{-K},$$

which is convergent as $k \to \infty$ because $\delta < 1$. Hence the series is convergent, and a steady state distribution exists. □