PRICE SETTING AND OPTIMAL MONETARY COOPERATION:

A NEO-KEYNESIAN PERSPECTIVE.

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Abstract

This paper analyses the implications of different price setting rules for optimal monetary cooperation. In doing so, we present a two-country dynamic general equilibrium model with imperfect competition, nominal price rigidities in which the export prices can be denominated either in the producer currency or in the consumer currency. In addition, the model can account both for efficient and inefficient shocks. Our main result is that the optimal cooperative policy crucially depends on the way prices are set. In particular, the paper gives some new insights on the optimal exchange rate regime and on the choice of the price index Central Banks should target. Moreover, this framework enables us to implement in a simple manner a welfare comparison of alternative policy rules in open economies.

Keywords: monetary policy rules, optimal monetary cooperation, exchange rate regime, pricing-to-market.

Classification JEL: E5, F4.

1 Introduction

In this paper, we try to assess the welfare costs of different policy rules both in the case of "producer-currency-pricing" and "local-currency-pricing". This work belongs to a large strand of literature that aims at analyzing monetary policy in open economies. Indeed, the new generation of dynamic general equilibrium models (DGEM) manages to mix simplicity with a rich behavioral structure. The framework we use here is closely related to those of Benigno and Benigno (2001) or Clarida, Gali Gertler (2002) among others: we present a two country model with imperfect competition and price rigidities in which efficient shocks (like technological shocks) coexist with inefficient ones (like "cost push" shocks). As opposed to previous studies, PPP does not hold here for two reasons: domestic bias in the household preferences and imperfect exchange rate pass-through. Benigno (1999) introduces "pricing-to-market" in a model quite similar to ours and Monacelli (2002) assumes that the consumer's preferences are biased towards locally produced goods, but neither pursues a welfare analysis. Nor do they derive the optimal monetary policy. In our model, we explicitly use an approximation of the welfare function in order to study the features of optimal monetary cooperation under commitment.

We show that the optimal outcome depends crucially on the price setting rules and on the kind of shocks that affects the economies. Here, we consider that firms can set their price in two different ways: the first one is the "producer-currency-pricing" (PCP) and the second one is the "local-currency-pricing" (LCP).

When prices are sticky in the producer's currency, we obviously find, in a slightly different model, the results of Benigno and Benigno (2001), and Clarida, Gali, Gertler (2002) about the optimal monetary policy (and more specially the optimal exchange rate regime) and about the gains from coordination. With efficient shocks, pure producer-price inflation targeting policies achieve the first best allocation. The nominal exchange rate is thus free to adjust to the required fluctuations of the terms of trade. As Milton Friedman (1953) advocated, it may be considered optimal in this context to have a flexible exchange rate regime. Nevertheless, when inefficient shocks hit the economies, the monetary authorities face a tradeoff between the inflation rate and the output gap stabilization: it is no longer possible to reach the first best allocation. So they cooperate optimally to adjust gradually the producer price levels, the output gaps and the terms of trade misalignments. In special cases, it may even be optimal to fix the exchange rate. As far as coordination gains are concerned, some non-negligible welfare improvements from cooperating exist even if the shocks are efficient.

These results are not robust to modifications of price setting. In particular, some properties can even be inverted in presence of "local-currency-pricing". First of all, no matter what kind of shocks affects the economies, the monetary authorities cannot manage to completely stabilize the producer inflation rates and the output gaps: there is always a tradeoff between the stabilization of import prices and producer prices. In order to gain some intuition on the optimal solution, it is worth restricting the model to the case of no preference bias. In this context, we find that the credible optimal monetary coordination under LCP is analogous to the one obtained under PCP when replacing the producer-price level by the consumer-price level, and the output gaps by the "consumption gaps". Therefore, following efficient shocks, it is now optimal to close the consumption gaps and to fully stabilize the consumer-price levels. Moreover, the predictions of the previous model about the optimal choice of an exchange rate regime are strongly modified by the assumption of LCP. The failure of the law of one price induced by the incomplete "pass-through" creates new incentives for the monetary authorities to control the exchange rate fluctuations. So, we prove that, following efficient shocks and with no preference bias, it is optimal to fix the exchange rate. Moreover, under a particular combination of parameters, the optimal solution implies a fixed exchange rate regime, independently from the nature of the shocks.

The paper is structured as follows. Section 2 presents the model and Section 3 proceeds to the log-linear approximation of the model and shows the reduced forms obtained in the PCP and LCP cases. Section 4 derives the optimal monetary cooperation under commitment. Section 5 computes the welfare costs of different monetary policy rules and analyses the transmission mechanisms.

2. The Model

The neo-Keynesian DGEMs that are abundantly used in the literature, are quite simple to derive and present relevant microeconomic foundations. They bring new perspectives in the field of monetary policy analysis due to their treatment of anticipations and the possibility to pursue welfare analysis.

Two symmetric countries, Home and Foreign, compose the world economy. In each country, there is a continuum of “single-good-firms” indexed on [0,1], producing differentiated goods that are imperfect substitute. The number of households is proportional to the number of firms. Consumers receive utility from consumption, real money holdings and disutility from labor. They are identical to each other in the sense that they share the same intertemporal elasticity of substitution and the same elasticity of labor supply with respect to the real wage. But, in each country, they have biased preferences towards locally produced goods. Household behavior consists in an intertemporal smoothing of consumption, an arbitrage between labor and consumption and a money demand. Financial markets are complete both domestically and internationally. In that context, we show that households are identical with respect to their consumption and labor supply choices. On the labor market, wages are fully flexible.

Firms are monopolistic competitors, produce differentiated products and set prices on a staggered basis *à la* Calvo (1983). Export prices are sticky either in the producer currency or in the buyer currency or in an intermediate situation.

Not only can the economies be affected by various “efficient” shocks like technological or demand shocks. But it is also possible to introduce inefficient shocks that lead to a short run inflation/output gap tradeoff for the conduct of monetary policy. In our model, we might rationalize those shocks as markup fluctuations in the labor market (due to wage rigidity for example) or as markup fluctuations in the goods market (following fiscal modifications).

2.1 Households

Consumer’s program

At time *t*, the utility function of a generic domestic consumer *h* belonging to country H is

(1)
$$U_t^h = E_t \sum_{s \geq t} \beta^{s-t} \left(U\left(C_{t+s}^h\right) + N\left(\frac{M(h)_{t+s}}{P_{t+s}}\right) - V\left(L_{t+s}^h\right) \right)$$

Households obtain utility from consumption of an aggregate index C_t^h and the liquidity services of holding money $\frac{M_t^h}{P_t}$, while receiving disutility from labor L_t^h.

Markets are complete both at international and domestic level. Each period, consumers can trade freely within a complete set of Arrow-Debreu securities. For each state of nature, there is a contingent one period nominal bond. We let $Q(s^{t+1} / s^{\to t})$ denote the price of this bond at date *t,* and $B_t^h(s^{t+1})$ the number of units bought by consumer *h*. The probability that the state of nature s^{t+1} occurs, knowing the past history of shocks until date *t*, is $\mu\left(s^{t+1} / s^{\to t}\right)$.

Each household *h* maximizes its utility function under the following budgetary constraint:

(2)
$$\sum_{s^{t+1}} \frac{Q(s^{t+1} / s^{\to t}) B_t^h(s^{t+1})}{P_t} + \frac{M_t^h}{P_t} = \frac{B_t^h(s^t)}{P_t} + \frac{M_{t-1}^h}{P_t} + \frac{W_t^h L_t^h}{P_t} + \frac{\pi_t^h}{P_t} - C_t^h + \frac{Transf_t^h}{P_t}$$

where W_t^h is the wage, π_t^h represents the nominal profits from the firm own by consumer *h* and $Transf_t^h$ are transfers between the government and the consumer.

The first order conditions corresponding to the quantity of contingent bonds are

(3) $$\forall s^{t+1} \quad Q(s^{t+1}/s^{\to t}) = \beta\mu\left(s^{t+1}/s^{\to t}\right)\frac{U_C\left(C_{t+1}^h\left(s^{t+1}\right)\right)}{U_C\left(C_t^h\right)}\frac{P_t}{P_{t+1}\left(s^{t+1}\right)}$$

A portfolio composed by one unit of each elementary security has the same value as a riskless one-period bond, so $\sum_{s^{t+1}} Q(s^{t+1}/s^{\to t}) = \frac{1}{1+i_t}$.

The two previous equations lead to the well-known Euler equation, reflecting the intertemporal consumption-smoothing behavior of households.

(4) $$U_C\left(C_t^h\right) = (1+i_t)\beta \underset{t}{E}\left(U_C\left(C_{t+1}^h\right)\frac{P_t}{P_{t+1}}\right)$$

We can show that, as financial markets are complete domestically, marginal utilities of consumption are equalized across households. As are the consumption levels since the utility function chosen here is additively separable.

Moreover, as in Erceg, Henderson and Levin (2000), each household is a monopoly supplier of a differentiated labor service. For the sake of simplicity, we assume that he sells his services to a perfectly competitive firm which transforms it into an aggregate labor input using the following technology

(5) $$L_t = \left[\int_0^1 L_t^{h\frac{\varepsilon_W - 1}{\varepsilon_W}} dh\right]^{\frac{\varepsilon_W}{\varepsilon_W - 1}}$$

The household faces a labor demand curve with constant elasticity of substitution:

(6) $$L_t^h = \left(\frac{W_t^h}{W_t}\right)^{-\varepsilon_W} L_t,$$

where $W_t = \left(\int_0^1 W_t^{h\,1-\varepsilon_W} dh\right)^{\frac{1}{1-\varepsilon_W}}$ is the aggregate wage rate.

The first order condition associated with wage setting is:

(7) $$\frac{\varepsilon_W}{\varepsilon_W - 1} V_L\left(L_t^h\right) = U_C\left(C_t\right)\frac{W_t^h}{P_t}$$

The real wage is equal to a constant markup over the marginal rate of substitution between consumption and labor. Because of wage flexibility, the households will all set the same wage and offer the same quantity of labor[1]:

$$L_t^h = L_t \text{ and } W_t^h = W_t \text{ pour tout } h \in [0,1] \text{ and pour tout } t.$$

Finally, the demand for real money holdings is:

(8) $$N_M\left(\frac{M_t^h}{P_t}\right) = U_C\left(C_t\right)\frac{i_t}{1+i_t}$$

The marginal rate of substitution between money and consumption is equal to the opportunity cost of holding it.

[1] Clarida, Gali, Gertler (2002) make the same assumption.

Demands for differentiated goods

Consumers prefer locally produced goods[2]. This composition bias in the household preferences determines the degree of openness at steady state. The aggregate consumption indexes are defined as follows

(9) $$C=\left[n^{\frac{1}{\xi}}C_H^{\frac{1-\xi}{\xi}}+(1-n)^{\frac{1}{\xi}}C_F^{\frac{1-\xi}{\xi}}\right]^{\frac{\xi}{1-\xi}} \quad \text{and} \quad C^*=\left[(1-n)^{\frac{1}{\xi}}C_H^{*\frac{1-\xi}{\xi}}+n^{\frac{1}{\xi}}C_F^{*\frac{1-\xi}{\xi}}\right]^{\frac{\xi}{1-\xi}},\ \xi>1.$$

C_H and C_F are consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F. ξ is the elasticity of substitution between bundles C_H and C_F. The elementary differentiated goods are imperfect substitutes with elasticity of substitution denoted ε.

(10) $$C_H=\left[\int_0^1 c(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right]^{\frac{\varepsilon}{\varepsilon-1}},\quad C_F=\left[\int_0^1 c(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } C_H^*=\left[\int_0^1 c^*(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right]^{\frac{\varepsilon}{\varepsilon-1}},\quad C_F^*=\left[\int_0^1 c^*(z)^{\frac{\varepsilon-1}{\varepsilon}}dz\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The consumption-based price indexes associated are defined as

(11) $$P=\left[nP_H^{1-\xi}+(1-n)P_F^{1-\xi}\right]^{\frac{1}{1-\xi}} \text{ et } P^*=\left[(1-n)P_H^{*1-\xi}+nP_F^{*1-\xi}\right]^{\frac{1}{1-\xi}}$$

with

(12) $$P_H=\left[\int_0^1 p(z)^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}},\quad P_F=\left[\int_0^1 p(z)^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}} \text{ and } P_H^*=\left[\int_0^1 p^*(z)^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}},\quad P_F^*=\left[\int_0^1 p^*(z)^{1-\varepsilon}dz\right]^{\frac{1}{1-\varepsilon}}.$$

Each household allocates consumption across the differentiated goods as follows

(13) $$\begin{cases}\forall z\in[0,n] & c(z)=\left(\dfrac{p(z)}{P_H}\right)^{-\varepsilon}C_H \quad c^*(z)=\left(\dfrac{p^*(z)}{P_H^*}\right)^{-\varepsilon}C_H^*\\ \forall z\in[n,1] & c(z)=\left(\dfrac{p(z)}{P_F}\right)^{-\varepsilon}C_F \quad c^*(z)=\left(\dfrac{p^*(z)}{P_F^*}\right)^{-\varepsilon}C_F^*\end{cases}$$

and

(14) $$\begin{cases}C_H=n\left(\dfrac{P_H}{P}\right)^{-\xi}C \quad C_H^*=(1-n)\left(\dfrac{P_H^*}{P^*}\right)^{-\xi}C^*\\ C_F=(1-n)\left(\dfrac{P_F}{P}\right)^{-\xi}C \quad C_F^*=n\left(\dfrac{P_F^*}{P^*}\right)^{-\xi}C^*\end{cases}.$$

Each producer faces the aggregate local and foreign demand given by

(15) $$\begin{cases}\forall h\in[0,1] & Y_H^d(h)=\left(\dfrac{p(h)}{P_H}\right)^{-\varepsilon}\left[\left(\dfrac{P_H}{P}\right)^{-\xi}nC+G\right],\ Y_H^{d*}(h)=\left(\dfrac{p^*(h)}{P_H^*}\right)^{-\varepsilon}\left(\dfrac{P_H^*}{P^*}\right)^{-\xi}(1-n)C^*\\ \forall f\in[0,1] & Y_F^{d*}(f)=\left(\dfrac{p(f)}{P_F}\right)^{-\varepsilon}\left[\left(\dfrac{P_F}{P}\right)^{-\xi}(1-n)C+G^*\right],\ Y_F^d(f)=\left(\dfrac{p^*(f)}{P_F^*}\right)^{-\varepsilon}\left(\dfrac{P_F^*}{P^*}\right)^{-\xi}nC^*\end{cases}$$

G and G^* are country-specific public expenditure shocks.

[2] As in Monacelli (2001).

Optimal risk sharing

In addition to the completeness of domestic markets, we also assume that financial markets are complete internationally. Households in both countries are allowed to trade in the contingent one-period nominal bonds denominated in the home currency. This leads to the following risk sharing condition [3]:

(16) $$\frac{U_C\left(C_t^*\right)}{U_C\left(C_t\right)}=\kappa\frac{S_tP_t^*}{P_t}$$

where S_t is the nominal exchange rate, and κ is a constant depending on initial conditions (here normalized to 1). Equation (16) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among state-contingent securities.

When markets are complete, it is no use evaluating the current account path in order to determine the relative consumption dynamics. Consumption levels in both countries differ only to the extent that the real exchange rate deviates from purchasing power parity. In our model, those deviations are allowed by two assumptions. The first one is the preference bias for locally produced goods, implying that real exchange rate depends on terms of trade. When $n=0{,}5$, this bias disappears. The second one is the possibility that prices might not be denominated in the producer currency, which generates failures of the law of one price.

Moreover, relation (16) ensures that in the model, consumption levels and the real exchange rate are stationary variables. Hence, should we consider stationary shocks, it would be relevant to analyze the model's properties in the neighborhood of a well-defined steady state.

2.2 Firms

Goods are produced with a technology that is linear in labor input as follows:

(17) $$\forall f\in[0,1],\ Y_t(f)=A_tL_t(f)\quad \text{et}\quad Y_t^*(f)=A_t^*L_t^*(f)$$

where A and A^* are exogenous technology parameters.

"Producer-currency-pricing" (PCP)

Firms are monopolist suppliers who set their price in the local currency. In this context, the law of one price holds:

$$\forall z\in[0,1]\quad p^*(z)S=p(z),\qquad P_F^*S=P_F,\qquad P_H^*S=P_H\ .$$

But, because of the preference bias, there is no purchasing power parity and

$$\frac{SP^*}{P}=\left(\frac{nT^{1-\xi}+1-n}{n+(1-n)T^{1-\xi}}\right)^{1/1-\xi}$$

where $T=\dfrac{SP_F^*}{P_H}$ denotes the terms of trade.

We assume that firms set prices according to the mechanism spelled out in Calvo (1983). In each period, a firm *h* (resp. *f*) faces a constant probability, $1-\alpha_H$ (resp. $1-\alpha_F^*$), of being able to reoptimize its nominal price. This probability is independent across firms and time in a same country. The average duration of a rigidity period is $1/1-\alpha_H$ (resp. $1/1-\alpha_F^*$). If a firm can reoptimize its price, it does so according to the following simple rule: $P_t(z)=\bar{\pi}_\bullet P_{t-1}(z)$ where $\bar{\pi}_\bullet$ is the stationary inflation rate.

[3] A full derivation of this result can be found in Chari et al (2000).

Therefore, the firm *h* chooses $\widetilde{P}_t(h)$ to maximize its intertemporal profit

(18) $$\underset{t}{E}\sum_{j=0}^{\infty}\alpha_H^j \Xi_{t,t+j} Y_{t+j}^d(h)\left[(1-\tau)\widetilde{P}_t(h)(\overline{\pi}_H)^j - MC_{t+j}P_{H,t+j}\right]$$

where $\Xi_{t,t+j} = \beta^j \dfrac{U_C(C_{t+j})P_t}{U_C(C_t)P_{t+j}}$ is the marginal value of one unit of money to the household, $MC_{t+j} = \dfrac{W_{t+j}}{P_{H,t+j}A_{t+j}}$ is the real marginal cost and $Y_{t+f}^d(h)$ is the aggregate demand addressed to the firm h. τ is a tax on firm's revenue. Due to our assumptions on the labor market, the real marginal cost is identical across producers. In our model, all firms that can reoptimize their price at time *t* choose the same level (see Woodford (1996) for example).

The first order condition associated with the firm's choice of $\widetilde{P}_t(h)$ is

(19) $$\underset{t}{E}\sum_{j=0}^{\infty}\alpha_H^j \Xi_{t,t+j} Y_{t+j}^d P_{H,t+j}\left[(1-\tau)\frac{\widetilde{P}_t(h)}{P_{H,t+j}}(\overline{\pi}_H)^j - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}\right] = 0 .$$

When the probability of being able to change prices tends towards unity, (19) implies that the firm sets its price equal to a constant markup, $\dfrac{\varepsilon}{(\varepsilon-1)(1-\tau)}$, over marginal cost. Otherwise the firm imposes this markup to the weighted average of marginal costs over time.

Only a fraction $(1-\alpha_H)$ of producers in country *H* can reoptimize its price, each period. So the aggregate producer-pric- index has the following dynamic:

(20) $$P_{H,t}^{1-\varepsilon} = \alpha_H P_{H,t-1}^{1-\varepsilon} + (1-\alpha_H)\widetilde{P}_t^{1-\varepsilon}(h)$$

Equations analogous to (18) and (19) hold for foreign producers.

"Local-currency-pricing" (LCP)

In our model, we only consider tradable goods. It would have been possible to introduce non-tradable goods as a source of deviation from PPP. But our assumption that export prices are set in the consumer currency, seems to be quite in line with a huge amount of empirical studies[4]. Those papers show in particular that fluctuations in the real exchange rate are principally caused by international market segmentation.

Now, each firm faces two distinct probabilities of being able to reoptimize its price for local sales and its price for sales in the foreign market. We denote α_H the probability prevailing in the local market (respectively α_F^* for producers of country *F*) and α_H^* the probability prevailing in the export market (respectively α_F for producers of country *F*).

First order conditions concerning the firm *h* become

(20) $$\underset{t}{E}\sum_{j=0}^{\infty}\alpha_H^j \Xi_{t,t+j} Y_{H,t+j}^d P_{H,t+j}\left[(1-\tau)\frac{\widetilde{P}_{H,t}(h)}{P_{H,t+j}}(\overline{\pi}_H)^j - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}\right] = 0$$

and $$\underset{t}{E}\sum_{j=0}^{\infty}(\alpha_H^*)^j \Xi_{t,t+j} Y_{H,t+j}^{d*} P_{H,t+j}\left[(1-\tau)\frac{\widetilde{P}_{H,t+j}^*(h)}{P_{H,t+j}^*}\frac{S_{t+j}P_{H,t+j}^*}{P_{H,t+j}}(\overline{\pi}_H)^j - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}\right] = 0 .$$

[4] Cf. Chari, Kehoe et McGrattan (2000) and Engel (1998) among others.

MC_{t+j} is the real marginal cost deflated by interior-production-price-index.

As seen before, the dynamics of price indexes are:

(21) $$P_{H,t}^{1-\varepsilon} = \alpha_H P_{H,t-1}^{1-\varepsilon} + (1-\alpha_H)\widetilde{P}_t^{1-\varepsilon}(h) \text{ and } P_{H,t}^{*\ 1-\varepsilon} = \alpha_H^* P_{H,t-1}^{*\ 1-\varepsilon} + (1-\alpha_H^*)\widetilde{P}_t^{*1-\varepsilon}(h).$$

Here again, we do not write down the analogous conditions for country *F*.

$CHGER_{H,t} = \frac{S_t P_{H,t}^*}{P_{H,t}}$ and $CHGER_{F,t} = \frac{P_{F,t}}{S_t P_{F,t}^*}$ represent the aggregate relative margins on export sales for producers in country *H* and *F* respectively.

Hybrid version

Both previous price-setting schemes are special cases of the following one. Here, we assume that a fraction of nominal exchange rate fluctuations automatically passes through import prices. That is to say that producers in both countries set $\hat{\widetilde{P}}_H^*(h)$ and $\hat{\widetilde{P}}_F(f)$ on a staggered basis so that the export prices are finally given by

(22) $$P_H^* = \hat{\widetilde{P}}_H S^{-\eta} \text{ and } P_F = \hat{\widetilde{P}}_F S^{\eta^*}.$$

The sticky components of export prices verify the following first order conditions

(23) $$\underset{t}{E}\sum_{j=0}^{\infty}(\alpha_H^*)^j \Xi_{t,t+j} Y_{H,t+j}^{d*} P_{H,t+j}\left[(1-\tau)\frac{\hat{\widetilde{P}}_{H,t+j}^*(h)}{\widetilde{P}_{H,t+j}^*}\frac{S_{t+j}P_{H,t+j}^*}{P_{H,t+j}}(\overline{\pi}_H)^j - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}\right] = 0$$

and $$\underset{t}{E}\sum_{j=0}^{\infty}\alpha_F^j \Xi_{t,t+j}^* Y_{F,t+j}^d(f) P_{F,t+j}^*\left[(1-\tau^*)\frac{\hat{\widetilde{P}}_F(f)}{\hat{\widetilde{P}}_{F,t+j}}\frac{P_{F,t+j}}{S_{t+j}P_{F,t+j}^*}(\overline{\pi}_F^*)^j - \frac{\varepsilon}{\varepsilon-1}MC_{t+j}^*\right] = 0.$$

3 Equilibrium

Market clearing conditions on goods market are obtained using the aggregator $\left[\int_0^1 \bullet^{\frac{\varepsilon-1}{\varepsilon}} dz\right]^{\frac{\varepsilon}{\varepsilon-1}}$:

(24) $$\begin{cases} (1-\omega)Y = \left(\frac{P_H}{P}\right)^{-\xi} nC + \left(\frac{P_H^*}{P^*}\right)^{-\xi}(1-n)C^* \\ (1-\omega^*)Y^* = \left(\frac{P_F}{P}\right)^{-\xi}(1-n)C + \left(\frac{P_F^*}{P^*}\right)^{-\xi} nC^* \end{cases}$$

where ω and ω^* are government consumption as a fraction of local production.

On the supply side, aggregate productions are linked to labor demands through the following relations

(25) $$\begin{cases} Y_{H,t} = \frac{A_t N_{H,t}}{V_{H,t}},\ Y_{H,t}^* = \frac{A_t N_{H,t}^*}{V_{H,t}^*} \\ Y_{F,t} = \frac{A_t N_{F,t}}{V_{F,t}},\ Y_{F,t}^* = \frac{A_t N_{F,t}^*}{V_{F,t}^*} \end{cases}$$

where $V_{\bullet,t} = \int_0^1 \left(\frac{P_t(h)}{P_{\bullet,t}} \right)^{-\varepsilon} dh \geq 1$ et $V_{\bullet,t}^* = \int_0^1 \left(\frac{P_t^*(f)}{P_{\bullet,t}^*} \right)^{-\varepsilon} df \geq 1$.

The $V_{\bullet,t}$ and $V_{\bullet,t}^*$ terms illustrate the dispersion of interior productions and exports for both countries.

Finally, the model is closed once the monetary policies are specified.

3.1 Log-linearization of the model.

The determinist steady state, around which we will linearize the model, is associated with the case where all shocks are held at their unconditional mean. There is no inflation and no depreciation. All price levels are equalized. In that context, PPP does hold and all macroeconomic aggregates are the same across countries.

We use identical utility function for all households in the world economy, given by

$$U_t^h = E_t \sum_{s \geq t} \beta^{s-t} \left(\frac{C_{t+s}^{1-\sigma}}{1-\sigma} + \chi \log \frac{M_{t+s}}{P_{t+s}} - K \frac{L_{t+s}^{1+\varphi}}{1+\phi} \right)$$

Different kind of shocks can be introduced in the model like

- technological shock a,
- money demand shock d,
- labor market markup shock μ^w,
- cost push shock u (stemming fiscal shock for example),
- government consumption shock g (denoting $g = -\ln(1-\omega)$).

In what follows, lower case letters stand for the logarithmic deviation from steady state.

First order conditions describing the household's behavior become

- Euler equations

(26) $$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}\left(i_t - E_t(\pi_{t+1})\right)$$

(27) $$c_t^* = E_t(c_{t+1}^*) - \frac{1}{\sigma^*}\left(i_t^* - E_t(\pi_{t+1}^*)\right)$$

- Wage equations

(28) $$w_t - p_t = \phi l_t + \sigma c_t + \mu_t^w$$

(29) $$w_t^* - p_t^* = \phi^* l_t^* + \sigma^* c_t^* + \mu_t^{w^*}$$

- Money demand equations

(30) $$m_t - p_t = \sigma c_t - \eta i_t + d_t$$

(31) $$m_t^* - p_t^* = \sigma^* c_t^* - \eta^* i_t^* + d_t^*$$

The optimal risk sharing condition can be written as

(32) $$\sigma c_t - \sigma^* c_t^* = chger_t = s_t + p_t^* - p\ .$$

Using equations (26), (27) and (32), it is easy to see that uncovered interest rate parity holds, independently from the specified price-setting rules:

(33) $$\underset{t}{E}\,\Delta s_{t+1} = i_t - i_t^* .$$

The real exchange rate is related to the terms of trade and the relative export margins by the following relation

(34) $$chger_t = (2n-1)\frac{t_t + t_t^*}{2} + \frac{chger_H - chger_F}{2}$$

where $t_t = p_{F,t} - p_{H,t}$, $t_t^* = p_{F,t}^* - p_{H,t}^*$ are the interior terms of trade in country *H* and *F*. $chger_{H,t} = s_t + p_{H,t}^* - p_{H,t}$ and $chger_{F,t} = p_{F,t} - s_t - p_{F,t}^*$ are of course the relative export margins of producers in country *H* and *F*.

Market clearing conditions (24) become

$$y_t = n(1-n)\xi\, t_t + n(1-n)\xi\, t_t^* + nc_t + (1-n)c_t^* + g_t$$

and $$y_t^* = -n(1-n)\xi\, t_t - n(1-n)\xi\, t_t^* + nc_t^* + (1-n)c_t + g_t^*,$$

which can be written, using (34), as

(35) $$y_t + y_t^* = c_t + c_t^* + g_t + g_t^* \quad \text{and} \quad y_t - y_t^* = Z\left(t_t + t_t^*\right) + (2n-1)\frac{chger_H - chger_F}{2\sigma} + g_t - g_t^*$$

with $Z = 2n(1-n)\xi + \dfrac{(2n-1)^2}{2\sigma}$.

A first order approximation of the production functions leads to:

(36) $$y_t = a_t + l_t \quad \text{and} \quad y_t^* = a_t^* + l_t^*$$

with

(37) $$y_t = n\, y_{H,t} + (1-n)\, y_{H,t}^* \quad \text{and} \quad y_t^* = n\, y_{F,t}^* + (1-n)\, y_{F,t} .$$

The $V_{\bullet,t}$ et $V_{\bullet,t}^*$ terms in (25) are constant up to a first order (cf. Erceg, Henderson et Levin (2000)).

3.2 The flexible price equilibrium

When prices are fully flexible, only efficient shocks - which means shocks that do not introduce new distortions - are relevant. Thus, in order to derive the flexible price allocation, markups in the goods market as well as in the labor market are assumed to be constant. By doing so, we do not want the natural level of output to reflect variations in the degree of efficiency of the economies. This property is all the more appropriate since we treat changes in the wage markup as standing for wage rigidity.

In the absence of price stickiness, the law of one price holds and firms set prices equal to a constant markup over marginal cost. It follows that $t_t = t_t^*$ and $chger_{H,t} = chger_{F,t} = 0$. The flexible allocation is therefore strictly independent from the price setting rules.

The flexible price model is closed by the addition of two price-setting relations to equations (26)-(37). Those conditions state that real marginal costs (in deviation from its steady state value) are equal to zero:

(38) $$w_t - p_{H,t} = a_t \quad \text{and} \quad w_t^* - p_{F,t}^* = a_t^* .$$

Reduced form of the flexible price model

The flexible price economies are hit by technological shocks and by government consumption shocks, $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ and $g_t = \rho_g g_{t-1} + \varepsilon_t^g$. The model is easily solved in terms of aggregate and relative variables. So, in the following, we denote for any variable X, $X^W = \frac{X + X^*}{2}$ and $X^R = \frac{X - X^*}{2}$.

Using equations (26)-(29), (32), (35), (36) and (38), we obtain the following reduced form:

Table 1: The flexible price model

$\bar{y}_t^W = \frac{1+\phi}{\sigma+\phi} a_t^W + \frac{\sigma}{\sigma+\phi} g_t^W$, $\bar{y}_t^R = \frac{(1+\phi)Z}{n+Z\phi} a_t^R + \frac{n}{n+Z\phi} g_t^R$	Aggregate and relative output
$\bar{c}_t^W = \frac{1+\phi}{\sigma+\phi} a_t^W - \frac{\phi}{\sigma+\phi} g_t^W$, $\bar{c}_t^R = \frac{(n-1/2)(1+\phi)}{\sigma(n+Z\phi)} a_t^R - \frac{(n-1/2)\phi}{\sigma(n+Z\phi)} g_t^R$	Aggregate and relative consumption
$\bar{t}_t = \frac{1+\phi}{n+Z\phi} a_t^R - \frac{\phi}{n+Z\phi} g_t^R$	Terms of trade
$\bar{r}_t^W = \sigma \underset{t}{E}\left(\Delta \bar{c}_{t+1}^W\right)$, $\bar{r}^R = \sigma \underset{t}{E}\left(\Delta \bar{c}_{t+1}^R\right)$	Average and relative interest rate
with $Z = 2n(1-n)\xi + \frac{(2n-1)^2}{2\sigma}$	

3.3 The "producer-currency-pricing" equilibrium

When prices are sticky, the economies are also hit by "cost-push" shocks following the stochastic processes $u_t = \rho_u u_{t-1} + \varepsilon_t^u$ and $u_t^* = \rho_u u_{t-1}^* + \varepsilon_t^u$.

Under PCP, the law of one price holds so that $t_t = t_t^*$ and $chger_{H,t} = chger_{F,t} = 0$. In this case, there is no sticky import price behavior and the supply side of the model is only composed by two Phillips curves.

The neo-Keynesian Phillips curves

When linearizing the first order condition (19), we obtain the following relation

(39) $$\tilde{p}_t(h) = \sum_{j=1}^{\infty} (\beta\alpha_H)^j \underset{t}{E}\left(\pi_{H,t+j}\right) + (1-\beta\alpha_H)\sum_{j=1}^{\infty} (\beta\alpha_H)^j \underset{t}{E}\left(mc_{t+j}\right)$$

where π_H is the production-price-index inflation rate.

Moreover, equation (20) becomes

(40) $$\pi_{H,t} = \left(\frac{1-\alpha_H}{\alpha_H}\right)\tilde{p}_t(h).$$

Combining the two preceding equations, we derive the so-called neo-Keynesian Phillips curve

(41) $$\pi_{H,t} = \beta \underset{t}{E} \pi_{H,t+1} + \lambda_H mc_t + u_{H,t} \text{ with } \lambda_H = \frac{(1-\alpha_H)(1-\beta\alpha_H)}{\alpha_H} \text{ and}$$

(42) $$mc_t = w_t - p_{H,t} - a_t$$

The same relation holds for country F, that is

(43) $$\pi^*_{F,t} = \beta^* \underset{t}{E}\pi^*_{F,t+1} + \lambda^*_F mc^*_t + u^*_{F,t} \text{ with } \lambda^*_F = \frac{(1-\alpha^*_F)(1-\beta\alpha^*_F)}{\alpha^*_F} \text{ and}$$

(44) $$mc^*_t = w^*_t - p^*_{F,t} - a^*_t .$$

The cost-push shocks that affect directly the inflation rates are defined by $u^*_{F,t} = \lambda^*_F u^*_t$ and $u_{H,t} = \lambda_H u_t$.

Reduced form of the PCP model

In the following a hat over a variable indicates the absolute deviation from its flexible price value. For example, $\hat{y}^W = y^W - \bar{y}^W$ is the world output gap.

After some algebra, the model collapses to a reduced form consisting in an aggregate demand equation, the uncovered interest rate parity, two Phillips curves and the relation defining the terms of trade. Besides, the marginal costs can be expressed easily in terms of the aggregate output gap and the terms of trade misalignments, using equations (42), (43), (28), (29), (35), (36) in deviation from the flexible price allocation.

Table 2: The PCP model

$\sigma \underset{t}{E}\left(\hat{y}^W_{t+1} - \hat{y}^W_t\right) = \frac{i_t + i^*_t}{2} - \bar{r}^W_t - \underset{t}{E}\pi^W_{t+1}$	ISw
$\underset{t}{E}\Delta s_{t+1} = i_t - i^*_t$	UIP
$\pi_{H,t} = \beta \underset{t}{E}\pi_{H,t+1} + \lambda_H\left[(\sigma+\phi)\hat{y}^W_t + (1/2+\phi Z)\hat{t}\right] + u_{H,t}$	ASH
$\pi^*_{F,t} = \beta \underset{t}{E}\pi^*_{F,t+1} + \lambda^*_F\left[(\sigma+\phi)\hat{y}^W_t - (1/2+\phi Z)\hat{t}\right] + u^*_{F,t}$	ASF
$\hat{t}_t = \hat{t}_{t-1} + \Delta s_t + \pi^*_{F,t} - \pi_{H,t} - \Delta\bar{t}_t$	TT
$\pi_t = n\pi_{H,t} + (1-n)\left(\Delta s_t + \pi^*_{F,t}\right)$	CPIH
$\pi^*_t = n\left(\pi_{H,t} - \Delta s_t\right) + (1-n)\pi^*_{F,t}$	CPIH

3.4 The incomplete "pass-through" model

The supply curves

When prices are set in the consumer currency, the aggregate supply curves are modified in the following way

(43) $$\pi_{H,t} = \beta \underset{t}{E}\pi_{H,t+1} + \lambda_H mc_t + u_{H,t} \quad \text{and} \quad \pi^*_{H,t} = \beta \underset{t}{E}\pi^*_{H,t+1} + \lambda^*_H\left(mc_t - chger_H\right) + u^*_{H,t}$$

(44) $$\pi^*_{F,t} = \beta^* \underset{t}{E}\pi^*_{F,t+1} + \lambda^*_F mc^*_t + u^*_{F,t} \text{ and } \pi_{F,t} = \beta^* \underset{t}{E}\pi_{F,t+1} + \lambda_F\left(mc^*_t - chger_F\right) + u_{F,t}$$

with $chger_{H,t} = chger_{H,t-1} + \Delta s_t + \pi^*_{H,t} - \pi_{H,t}$ and $chger_{F,t} = chger_{F,t-1} + \pi_{F,t} - \Delta s_t - \pi^*_{F,t}$.

More generally, if producers pass only a fraction of the nominal exchange rate variations on to export prices, according to the mechanism presented in the previous section, the export-price inflation rates will be given by the following equations

(45) $$\pi^*_{H,t} = \tilde{\pi}_{H,t} - \eta\Delta s_t \quad \text{and} \quad \tilde{\pi}_{H,t} = \beta \underset{t}{E}\tilde{\pi}_{H,t+1} + \lambda^*_H\left(mc_t - chger_H\right) + u^*_{H,t}$$

(46) $\pi_{F,t} = \tilde{\pi}_{F,t} + \eta^* \Delta s_t$ and $\tilde{\pi}_{F,t} = \beta^* \underset{t}{E}\, \tilde{\pi}_{F,t+1} + \lambda_F \left(mc_t^* - chger_F \right) + u_{F,t}$.

The cost-push shocks, here again, are defined by $u_{H,t} = \lambda_H u_t$, $u_{H,t}^* = \lambda_H^* u_t$, $u_{F,t}^* = \lambda_F^* u_t^*$ and $u_{F,t} = \lambda_F u_t$.

Reduced form of the incomplete "pass-through" model

As we have already done for the PCP model, it is possible to considerably simplify the model, which leads to the reduced form presented in table 3. The number of state variables has increased: The deviations from the law of one price imply that from now on, relative export margins must be taken into account and the interior terms of trade are not equalized across countries anymore.

Table 3: Incomplete "pass-through" model

$\sigma \underset{t}{E}\left(\hat{y}_{t+1}^W - \hat{y}_t^W \right) = \dfrac{i_t + i_t^*}{2} - \bar{r}_t^W - \underset{t}{E}\, \pi_{t+1}^W$	ISw
$\underset{t}{E}\, \Delta s_{t+1} = i_t - i_t^*$	UIP
$\pi_{H,t} = \beta \underset{t}{E}\, \pi_{H,t+1} + \lambda_H mc_t + u_{H,t}$	ASH
$\tilde{\pi}_{H,t} = \beta \underset{t}{E}\, \tilde{\pi}_{H,t+1} + \lambda_H^* \left(mc_t - chger_{H,t} \right) + u_{H,t}^*$	ASH*
with $mc_t = (\sigma + \phi)\hat{y}_t^W + \left(\dfrac{1}{2} + \phi Z \right)\left(\dfrac{\hat{t}_t + \hat{t}_t^*}{2} \right) + (2n-1)\left(\dfrac{\hat{t}_t^* - \hat{t}_t}{4} \right) + \dfrac{(2n-1)}{4\sigma}\phi\left(chger_{H,t} - chger_{F,t} \right) + \dfrac{chger_{H,t}}{2}$	
$\pi_{F,t}^* = \beta^* \underset{t}{E}\, \pi_{F,t+1}^* + \lambda_F^* mc_t^* + u_{F,t}^*$	ASF
$\tilde{\pi}_{F,t} = \beta^* \underset{t}{E}\, \tilde{\pi}_{F,t+1} + \lambda_F \left(mc_t^* - chger_{F,t} \right) + u_{F,t}$	ASF*
with $mc_t^* = (\sigma + \phi)\hat{y}_t^W - \left(\dfrac{1}{2} + \phi Z \right)\left(\dfrac{\hat{t}_t + \hat{t}_t^*}{2} \right) + (2n-1)\left(\dfrac{\hat{t}_t^* - \hat{t}_t}{4} \right) - \dfrac{(2n-1)}{4\sigma}\phi\left(chger_{H,t} - chger_{F,t} \right) + \dfrac{chger_{F,t}}{2}$	
$\hat{t}_t = \hat{t}_{t-1} + \tilde{\pi}_{F,t} + \eta \Delta s_t - \pi_{H,t} - \Delta \bar{t}_t$	TT
$\hat{t}_t^* = \hat{t}_{t-1}^* + \Delta s_t + \pi_{F,t}^* + \eta^* \Delta s_t - \tilde{\pi}_{H,t} - \Delta \bar{t}_t$	TT*
$chger_{H,t} = chger_{H,t-1} + (1-\eta)\Delta s_t + \tilde{\pi}_{H,t} - \pi_{H,t}$	
$chger_{F,t} = chger_{F,t-1} + \tilde{\pi}_{F,t} - (1-\eta^*)\Delta s_t - \pi_{F,t}^*$	
$\pi_t = n\pi_{H,t} + (1-n)\pi_{F,t}$	CPIH
$\pi_t^* = n\pi_{F,t}^* + (1-n)\pi_{H,t}^*$	CPIF

The hybrid price setting scheme has the advantage to link continuously the PCP and the LCP models. When $\eta = \eta^* = 1$ and when the degrees of rigidity that one producer faces are the same across markets (i.e. $\alpha_H = \alpha_H^*$ and $\alpha_F = \alpha_F^*$), the model above is equivalent to the PCP model. But, with $\eta = \eta^* = 0$, we obtain the LCP model. Actually, in order to gain further intuition on the transmission mechanisms at work under "local-currency-pricing", it is worth focusing on the following particular case.

LCP model without preference bias and with similar degree of rigidity on import and local markets.

As it was highlighted by Smets and Wouters (2001), the degree of price rigidity affecting the import goods is likely to be very close to the one concerning domestic sales of locally produced goods. Therefore, we believe that the assumption consisting in $\alpha_F^* = \alpha_H^*$ and $\alpha_F = \alpha_F$ might be justified in the LCP model. Combined with the absence of preference bias (i.e. $n = 1/2$), this assumption permits to derive a highly tractable reduced form of the LCP model.

Table 4: LCP model

$\sigma \underset{t}{E}\left(\hat{y}_{t+1}^W - \hat{y}_t^W\right) = \frac{i_t + i_t^*}{2} - \bar{r}_t^W - \underset{t}{E}\pi_{t+1}^W$	ISw
$\underset{t}{E}\Delta s_{t+1} = i_t - i_t^*$	UIP
$\pi_t = \beta \underset{t}{E}\pi_{t+1} + \lambda_H\left[(\sigma+\phi)\hat{y}_t^W + chger\right] + \bar{u}_t$	CASH
$\pi_t^* = \beta \underset{t}{E}\pi_{t+1}^* + \lambda_F\left[(\sigma+\phi)\hat{y}_t^W - chger\right] + \bar{u}_t^*$	CASF
$chger_t = chger_{t-1} + \Delta s_t + \pi_t^* - \pi_t$	RER

The cost-push shocks that impact the consumer-price inflation rates are given by $\bar{u}_t = \lambda_H\left(u_t + u_t^*\right)$ and $\bar{u}_t^* = \lambda_F\left(u_t + u_t^*\right)$.

4 Optimal monetary cooperation under commitment

4.1 Second order expansion of the aggregate welfare

If the monetary authorities accept to cooperate, they will maximize the unconditional expectation of the unweighted average of household utility functions. Since the consumption levels and working hours are equalized across households, the global welfare function is given by

$$W = \underset{0}{E}\sum_{t=0}^{\infty}\beta^t\omega_t \text{ where } \omega_t = \frac{U(C_t)+U(C_t^*)}{2} - \frac{V(L_t)+V(L_t^*)}{2}.$$

In this specification, the utility gains derived from the liquidity service of holding money has been assumed to be very small. We then take a second order approximation of this welfare function around a steady state in which a taxation subsidy completely offsets the monopolistic distortions in both countries[5]. In this context, the flexible price allocation is the first best solution.

So, by neglecting terms independent from monetary policy, we obtain that

(47) $$W = -\frac{1}{2}U_C\bar{C}\underset{0}{E}\sum_{t=0}^{\infty}\beta^t\Lambda_t ,$$

where

$$\Lambda_t = (\sigma+\phi)\left(\hat{y}_t^W\right)^2 + n(1-n)\xi\left(\frac{\hat{t}_t^2 + \hat{t}_t^{*2}}{2}\right) + \sigma\left(\hat{c}_t^R\right)^2 + \phi\left(\hat{y}_t^R\right)^2 + \frac{\varepsilon}{2}\left(n\frac{\pi_{H,t}^2}{\lambda_H} + (1-n)\frac{\tilde{\pi}_{H,t}^2}{\lambda_H^*}\right) + \frac{\varepsilon}{2}\left(n\frac{\left(\pi_{F,t}^*\right)^2}{\lambda_F^*} + (1-n)\frac{\tilde{\pi}_{F,t}^2}{\lambda_F}\right)$$

[5] The full derivation of the welfare quadratic expansion is analogous to the one found in Benigno (2001).

Under the "producer-currency-pricing" assumption, the instantaneous welfare approximation is given by

$$\Lambda_t = (\sigma+\phi)\left(\hat{y}_t^W\right)^2 + Z(1/2+\phi Z)\hat{t}_t^2 + \frac{\varepsilon}{2\lambda_H}\pi_{H,t}^2 + \frac{\varepsilon}{2\lambda_F^*}\left(\pi_{F,t}^*\right)^2 \tag{48}$$

This expression is to be related to the work of Benigno and Benigno (2001). When $n=1/2$ (that is to say when the preference bias vanishes), we find the same approximated welfare function. World output gap fluctuations affect welfare both through the consumption and the labor supply channels. Moreover, terms of trade misalignments are costly due to its expenditure-switching effects and its impact on relative labor supply. The distortions associated with the price rigidities are captured by the inflation rate variances whose weights are functions of the degrees of rigidity and the markup on goods market.

To get a better intuition on the objective function under incomplete "pass-through", let us consider the case of "local-currency-pricing" and no preference bias. Λ_t is then given by

$$\Lambda_t = (\sigma+\phi)\left(\hat{y}_t^W\right)^2 + \frac{\xi}{4}\left(\frac{\hat{t}_t^2+\hat{t}_t^{*2}}{2}\right) + \phi\frac{\xi^2}{4}\left(\frac{\hat{t}_t+\hat{t}_t^*}{2}\right)^2 + \frac{(chger_t)^2}{4\sigma} + \frac{\varepsilon}{4}\left(\frac{\pi_{H,t}^2}{\lambda_H} + \frac{\tilde{\pi}_{H,t}^2}{\lambda_H^*}\right) + \frac{\varepsilon}{4}\left(\frac{\left(\pi_{F,t}^*\right)^2}{\lambda_F^*} + \frac{\tilde{\pi}_{F,t}^2}{\lambda_F}\right)$$

The welfare costs due to deviations from the law of one price enter the objective function through the variance of the real exchange rate. Besides, the relative cost of import price and production price rigidities depends on the stationary openness ratio and on the relative degree of rigidity.

4.2 Derivation of the optimal policy under commitment

The general case

The optimal monetary cooperation under commitment is derived by maximizing the welfare function (47) under the structural equilibrium conditions ASH, ASH*, ASF, ASF*, REMH, REMF, TT and TT*. The interest rates are then determined by the aggregate demand equations.

The optimal plan can be described using the Lagrangian method with multipliers μ_H, μ_H^*, μ_F^* and μ_F associated with the aggregate supply equations, ν and ν^* with the terms of trade equations, θ and θ^* with the relative export margin equations.

The first-order necessary conditions with respect to the inflation rates are

$$n\varepsilon\pi_{H,t} = \Delta\mu_{H,t} + \lambda_H\nu_t + \lambda_H\theta_t\ , \tag{49}$$

$$(1-n)\varepsilon\pi_{H,t}^* = \Delta\mu_{H,t}^* + \lambda_H^*\nu_t^* - \lambda_H^*\theta_t\ , \tag{50}$$

$$n\varepsilon\pi_{F,t}^* = \Delta\mu_{F,t}^* - \lambda_F^*\nu_t^* + \lambda_F^*\theta_t^*\ , \tag{51}$$

$$(1-n)\varepsilon\pi_{F,t} = \Delta\mu_{F,t} - \lambda_F\nu_t - \lambda_F\theta_t^*\ . \tag{52}$$

After some algebra, those corresponding to the world output gap and the interior terms of trade, lead to

$$2\hat{y}_t^W = -\mu_{H,t} - \mu_{H,t}^* - \mu_{F,t}^* - \mu_{F,t}\ , \tag{53}$$

$$2\hat{y}_t^R = -\mu_{H,t} - \mu_{H,t}^* + \mu_{F,t}^* + \mu_{F,t} + \frac{4}{1+2\phi Z}\left(\nu_t^W - \beta\,\underset{t}{E}\,\nu_{t+1}^W\right), \tag{54}$$

$$n(1-n)\xi\,\hat{t}_t^R + (n-1/2)\hat{y}_t^W = \hat{\nu}_t^R - \beta\,\underset{t}{E}\,\hat{\nu}_{t+1}^R\ . \tag{55}$$

Finally, the conditions associated with the relative export margins and the nominal exchange rate imply that

(56) $$\frac{\mu_{H,t} + \mu^*_{F,t} - \mu^*_{H,t} - \mu_{F,t}}{2} = \theta^W_t - \beta \underset{t}{E}\, \theta^W_{t+1},$$

(57) $$\frac{\hat{c}^R_t}{2} = \frac{\mu^*_{F,t} + \mu^*_{H,t} - \mu_{F,t} - \mu_{H,t}}{4} + \theta^R_t - \beta \underset{t}{E}\, \theta^R_{t+1} - \frac{(2n-1)\phi}{\sigma(1+2\phi Z)}\left(\nu^W_t - \beta \underset{t}{E}\, \nu^W_{t+1}\right),$$

(58) $$\eta \nu_t + \eta^* \nu^*_t + (1-\eta)\theta_t - (1-\eta^*)\theta^*_t = 0\,.$$

The first implication of the optimal stabilization plan is summarized in the following result.

Result 1 *Under incomplete « pass-through », i.e.* $1 > \eta, \eta^* \geq 0$*, the optimal cooperative policy cannot achieve the first best allocation. Independently from the kind of shock affecting the economies, the monetary authorities always face a tradeoff between import-price and interior-production-price stabilization. The optimal plan requires adjusting gradually the price levels and the nominal exchange rate.*

Corollary 1 *Provided shocks are drawn from stationary distributions, prices and nominal exchange rate are stationary variables.*

The « producer-currency-pricing » case

Under PCP, the derivation of the optimal policy is easier because of the restricted number of state variables. The welfare function (48) has to be maximized under only two constraints: domestic and foreign Phillips curves. Equation TT determines residually the exchange rate, while the aggregate demand equations give the optimal paths of nominal interest rates. Here again we use the Lagrangian method and denote μ and μ^* the multipliers.

The first order conditions with respect to $\hat{y}^W$, $\hat{t}$, π_H and π^*_F are given by

(59) $$2Z\hat{t}_t = \mu^*_t - \mu_t\,,\ \hat{y}^W_t = -\frac{\mu_t + \mu^*_t}{2}\,,\ \varepsilon \pi_{H,t} = \mu_t - \mu_{t-1} \text{ and } \varepsilon \pi^*_{F,t} = \mu^*_t - \mu^*_{t-1}\,.$$

Using (35), it is possible to show that the optimal plan is characterized by the two following equations

(60) $$\varepsilon \pi_{H,t} = -\hat{y}_t + \hat{y}_{t-1} \text{ and } \varepsilon \pi^*_{F,t} = -\hat{y}^*_t + \hat{y}^*_{t-1}\,.$$

Those relations can also be derived from equations (49) to (58) by simply posing $\eta = \eta^* = 1$, $\alpha_H = \alpha^*_H$ and $\alpha_F = \alpha^*_F$. This leads to equations analogous to (59) where μ and μ^* are replaced by $\mu_H + \mu^*_H$ and $\mu_F + \mu^*_F$.

Under the "producer-currency-pricing" assumption, we revisit the results obtained by Benigno and Benigno (2001) in a slightly different framework.

Result 2 *Following efficient shocks, the optimal cooperative policy achieves the flexible price allocation if and only if prices are denominated in the producer currency. In that case, pure inflation targeting policies implement the optimal solution.*

Result 3 *Following inefficient shocks, the monetary authorities face an inflation/output gap tradeoff so that, in the optimal cooperative solution, the producer inflation rates are state contingent and it is no longer possible to fully stabilize the economies.*

« Local-currency-pricing » without preference bias

In order to gain some intuition on the optimal monetary cooperation when prices are denominated in the consumer currency, let's assume that $n = 0{,}5$. In that context, the real exchange rate moves only to the extent that there are some deviations from the law of one price. Furthermore, we force the degrees of rigidity to be the same within each country (i.e. $\alpha_H = \alpha_F$ et $\alpha_H^* = \alpha_F^*$).

Combining equations (49) to (58), it is straight forward to show that the optimal policy can be described by the two following equations

(61) $$\varepsilon \pi_t = -\hat{c}_t + \hat{c}_{t-1} \text{ and } \varepsilon \pi_t^* = -\hat{c}_t^* + \hat{c}_{t-1}^*$$

The two relations are similar to those obtained in the PCP model where the producer-price inflation rates and the output gaps are replaced by the consumer-price inflation rates and the "consumption gaps".

Result 4 *Without preference bias and when* $\alpha_H = \alpha_F$ *and* $\alpha_H^* = \alpha_F^*$*, the optimal cooperative policies in the* ***PCP*** *and in the* ***LCP*** *models are analogous but do not target the same objectives.*

PCP	***LCP***
$\varepsilon \pi_{H,t} = -\hat{y}_t + \hat{y}_{t-1}$	$\varepsilon \pi_t = -\hat{c}_t + \hat{c}_{t-1}$
$\varepsilon \pi_{F,t}^* = -\hat{y}_t^* + \hat{y}_{t-1}^*$	$\varepsilon \pi_t^* = -\hat{c}_t^* + \hat{c}_{t-1}^*$

When prices are set in the consumer currency, it is optimal to adjust the consumer price level to the variation of the "consumption gap" where as, under producer-currency-pricing, the producer price inflation rate is linked to the output gap fluctuations.

Corollary 2: *Under "local-currency-pricing", with no preference bias and following efficient shocks, the optimal solution consists in completely stabilizing the consumer-price levels and closing the consumption gaps. Pure consumer-price inflation targeting implements the optimal policy.*

4.3 Cooperation gains and implementation of the optimal policy

In the model developed in this paper, the absence of cooperative gains is only validated in some particular cases. As Benigno and Benigno (2001a) emphasized, when prices are denominated in the producer currency, the cooperative optimal plan leads to the same allocation as the Nash equilibrium, following efficient shocks and with, $\xi = 1$ and $n = 1/2$. Actually, the decentralized plan achieves the first best solution so that there is no gain to expect from any cooperation.

In general, the gains are likely to be all the more substantial as inefficient shocks hit the economies or as exchange rate "pass-through" is incomplete. The underlying reason is that the perceived tradeoffs are more favorable at the central-planner level.

As far as the policy implementation is concerned, Benigno and Benigno (2001b) gives an interesting result that can be easily extended to our framework under "producer-currency-pricing". They show that it is possible to replicate the optimal outcome by assigning to decentralized monetary authorities some well-defined "flexible" inflation targeting rules. Each policymaker commits to a loss function penalizing for the deviation of the inflation rate from target and for changes in the output gap.

However, such a result doesn't hold under incomplete "pass-through". It is impossible to implement the optimal solution without some kind of exchange rate arrangement.

4.4 Optimal exchange rate regime

Under "producer-currency-pricing"

Following efficient shocks, the optimal policy replicates the flexible price allocation. Therefore, as the inflation rates are equal to zero and the "terms of trade gap" is closed, equation TT shows that the nominal exchange rate has to adjust to the required terms of trade path under flexible prices. This property seems to plead in favor of a flexible exchange rate regime when only real shocks prevail.

However, in presence of inefficient shocks, the associated inflation/output gap tradeoff does not allow to fully stabilize the economies. The optimal monetary cooperation targets the producer price levels and the nominal exchange rate. A fixed exchange rate regime might even be optimal if $\varepsilon = 2Z$.

Indeed, combining equations (60) and TT, it is easy to show that the inflation rate differential realizes exactly the required terms of trade adjustment if $\varepsilon = 2Z$, leaving no role for exchange rate variations.

Result 5 *Under PCP and following efficient shocks, it is optimal to let the exchange rate freely adjust to the efficient fluctuations of international relative prices. But, following inefficient shocks, an exchange rate management is needed and it is optimal to fix it when* $\varepsilon = 2Z$ *.*

Under « local-currency-pricing »

In the presence of some kind of « pricing-to-market », the law of one price does not hold and it may seem quite appropriate to limit exchange rate variations in order to minimize the welfare costs associated with these distortions.

The following result shows that a fixed exchange rate regime is optimal under certain conditions.

Result 6 *Under LCP and with* $n = 0{,}5$ *,* $\alpha_H = \alpha_F$ *,* $\alpha_H^* = \alpha_F^*$ *, the optimal cooperative policy imposes a fixed exchange rate regime if either shocks are efficient or* $\varepsilon\sigma = 1$.

Following efficient shocks, the optimal policy fully stabilizes the consumer price levels and closes the consumption gaps. Furthermore, using equations CASH and CASF, we see that the real exchange rate remains constant. So equation RER implies that the nominal exchange rate is fixed.

Otherwise, reminding that the purchasing power parity holds in the flexible equilibrium without preference bias, we make use of equations (61) to show that the optimal real exchange rate variations are matched by the inflation rate differential if $\varepsilon\sigma = 1$.

5 Welfare comparisons of alternative monetary policy rules

In this section, we compare the properties of 6 sets of monetary policy rules:

1) Pure consumer-price inflation targeting **(IPC)**
2) Pure interior-production-price inflation targeting **(IPPI)**
3) Taylor rules targeting the consumer-price inflation with a weight of 1,5 and the output gap with a weight of 0,5 **(TaylIPC)**
4) Taylor rules targeting the interior-producer-price inflation with a weight of 1,5 and the output gap with a weight of 0,5 **(TaylPQI)**

5) Symmetric fixed exchange rate policy **(FIXE)**

6) The optimal cooperative policy **(OPT)**

The symmetric fixed exchange rate policy defined in 6) is implemented by the following rules

$$i_t = 1{,}5\,\pi_{H,t} + 0{,}5\,\hat{y}_t + \frac{b}{1-b} s_t \text{ and } i_t^* = 1{,}5\,\pi_{F,t}^* + 0{,}5\,\hat{y}_t^* - \frac{b}{1-b} s_t$$

where *b* tends towards 1.

5.1 Calibration of the model

We try to make the model representative of the interactions between the United States (US) and the euro zone (EZ). The main differences in the calibration of both country models concern the degrees of rigidity. Following recent empirical works on the estimation of forward-looking aggregate supply equations, such as Gali et al. (2001a), we assume that α_H and α_F^* are 0.66 and 0.75, which implies that the durations of contracts are 3 and 4 quarters in US and Europe, respectively. These authors noticed that, without inflation inertia, the estimated degrees of rigidity are too high. As a consequence, by choosing the appropriate durations of contracts, we may overestimate the parameters λ_H, λ_F^* and then underestimate the welfare costs.

Then we choose $\sigma = \sigma^* = 1$ and $\phi = \phi^* = 5$. Note that the unitary intertemporal elasticity of substitution makes the model consistent with a balance growth path. In particular, the long-term labor supply does not depend on trends on the technological variables. Besides, these values are taken by Gali et al (2001b) in their baseline calibration.

Finally, we pose $\xi = 2$, n=0,7 and $\varepsilon = 7$. The stationary openness ratio in both economies is 30%. And the markup in the goods market is close to 10% (as did Rotemberg et Woodford (1999)). The choice of the intratemporal elasticity of substitution ξ is much more difficult. RBC literature assumes values in the rage of 1-2 whereas some recent studies[6] find this parameter closer to 6.

Two kinds of shocks are examined in this section: technological shocks and cost-push shocks affecting EZ exclusively.

5.2 Transmission of shocks

A positive technological shock in EZ (Appendix 1)

When prices are set in the producer currency, the optimal policy fully stabilizes the output gaps and the production price levels. The nominal exchange rate is free to adjust and here, euro depreciates on impact, leading to an improvement of the European terms of trade, and then appreciates till reaching its initial level. Consumer prices fluctuate strongly due to the one-to-one pass-through of nominal exchange rate. Finally, the interest rates are accommodating in both countries, the decrease being stronger in Europe.

Among the different policy rules, the Taylor rules, incorporating a producer-price inflation objective, lead to simulations qualitatively close to the optimal paths as far as output gaps, consumer-price inflation rates and terms of trade are concerned. However, the nominal exchange rate is not stationary and appreciates in the long run. Neither is it stationary with the CPI-targeting Taylor rules. However, fixed exchange rate policy and pure CPI targeting bring back the exchange rate to its initial level. Notice that terms of trade are hump-shaped under rules incorporating some kind of exchange rate arrangement.

[6] Cf. Harrigan (1993) and Trefler et Lai (1999) among others.

Under "local-currency-pricing", the optimal cooperative policy cannot achieve the first best allocation. Output in EZ goes below its natural level whereas the output gap is positive in the US. The CPI inflation rates are smoother than in the PCP case. Interest rates moves in opposite directions: accommodating in Europe and restricting in the US. Exchange rate depreciates more strongly than under the "producer-currency-pricing" assumption.

The pure CPI targeting policy has the same properties as the optimal one since it implements the optimal outcome when there is no preference bias. Other policies lead to similar output gap and terms of trade simulations but fail to reproduce the optimal paths of prices and nominal exchange rate.

Actually, the hump-shaped response of the terms of trade seems to be independent of monetary policy. This observation is partially corroborated by the following result (that we also find in Benigno (1999)).

Result 7 *Under the assumptions of Corollary 2, the terms of trade are independent from monetary policy.*

Combining equations CASH and CASF to equations TT and TT*, one easily proves this result.

It is worth noting that, even if prices are sticky in the consumer currency, a depreciation does not imply a deterioration of the terms of trade. This observation answers to a common criticism addressed to LCP sticky price models which are "accused" of making counter-factual terms of trade response to exchange rate[7].

Cost-push shock in EZ (Appendix 2)

Under "producer-currency-pricing", the optimal policy cannot replicate the flexible price equilibrium. Output gap in the euro zone deteriorates and there is an excess demand in the US. Euro appreciates and terms of trade improve initially. Interest rates increase in both countries.

The Taylor rules imply impulse-responses quite different from the optimal simulations, in particular for the nominal exchange rate and the European CPI. The pure inflation targeting strategies give better results.

When prices are set in the consumer currency, the optimal cooperative policy now leads to a slight deterioration of the US output gap. Moreover, the variations of the European CPI inflation rate are inverted with respect to the PCP case, the US interest rate decreases on impact and euro appreciates more strongly.

As for the technological shock, pure CPI targeting gives the best outcome. The Taylor rules lead to unsatisfactory responses for exchange rate and consumer prices.

5.3 Welfare analysis

We now proceed to a quantitative welfare assessment of the different monetary policies presented previously. Technological and cost-push shocks are treated separately. For each policy, we compute the aggregate utility function

$$W = -\frac{1}{2} U_C \overline{C} \, E\left[\underset{0}{E} \sum_{t=0}^{\infty} \beta^t \Lambda_t \right]$$

[7] Cf. Obstfeld and Rogoff (2000)

As in Woodford (1999), the unconditional mean operator is taken over the initial conditions of the exogenous variables which follow the stationary distribution of its stochastic process.

The welfare cost of any monetary policy is measured by the index δ :

$$\delta^j = -(1-\beta)\frac{W^j}{U_C\overline{C}}$$

δ^j represents the permanent shift in steady state consumption under policy *j*.

Table 1 in Appendix 3 shows obviously that the introduction of incomplete "pass-through" induces welfare costs and enhances the cooperative gains. Moreover, the pure inflation targeting rules perform much better than other rules since those policies embeds some feature of the optimal outcome.

As far as Taylor rules are concerned, CPI targeting seems to be better suited than IPPI targeting, in particular under "local-currency-pricing".

Finally, the symmetric fixed exchange rate policy gives relatively good results under inefficient shocks.

6 Conclusion

The main result of this paper is that the international optimal monetary cooperation depends crucially on the way prices are set. We show that the introduction of "pricing-to-market" changes previous results found in the literature. Under "local-currency-pricing", the monetary authorities should target the consumer price index instead of the producer price index. Furthermore, the properties of the optimal exchange regime are almost inverted, compared with what we derived in the "producer-currency-pricing" case.

The simulations and the welfare analysis presented in the last section of this paper only intend to illustrate the importance of price setting assumptions. In order to obtain more realistic and consistent calculations, a calibration exercise of the shocks should be pursued and we might have to extend the model to incorporate incomplete markets and wage rigidity. We let this work for future research.

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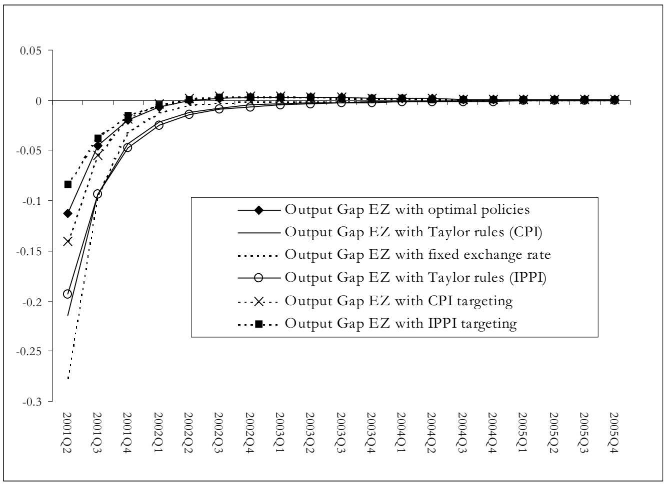
Appendix: Transmission mecanisms and Welfare analisis of different monetary policies.

1. Asymmetric technological shock

Euro zone benefits from a positive technological shock of 1 GDP point on one quarter with a persistence coefficient of 0.8.

PTM



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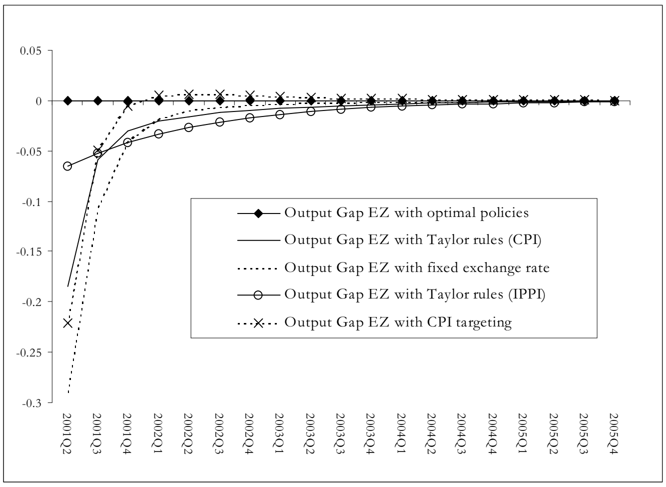

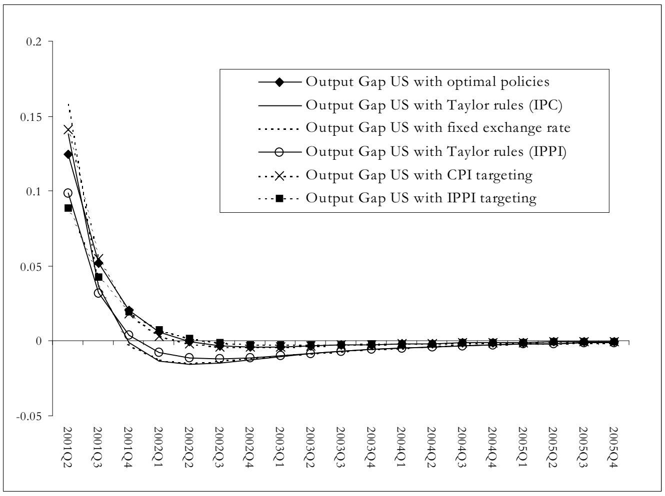

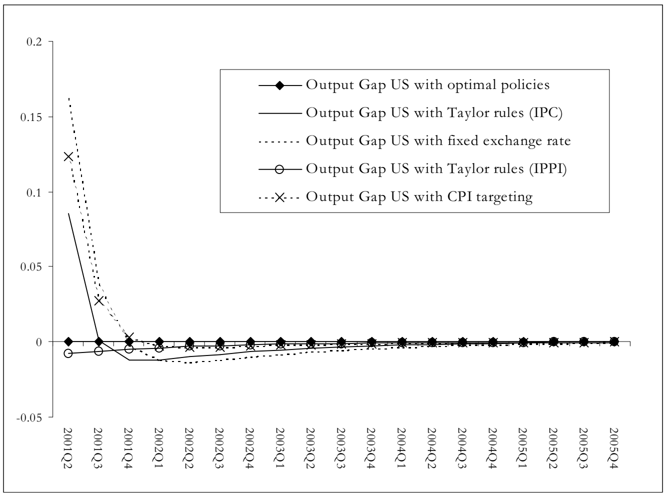

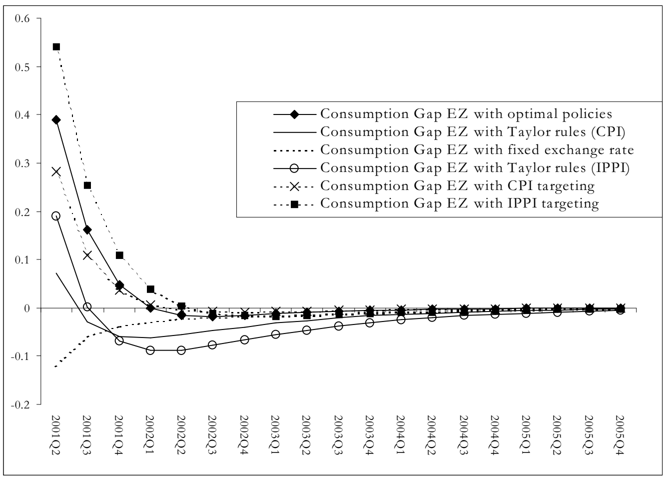

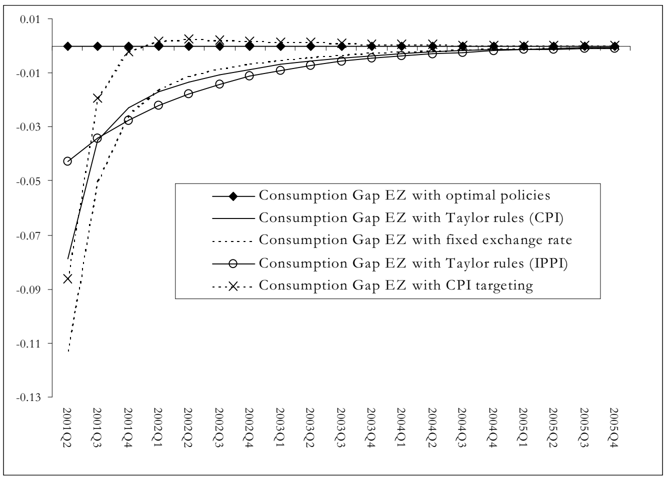

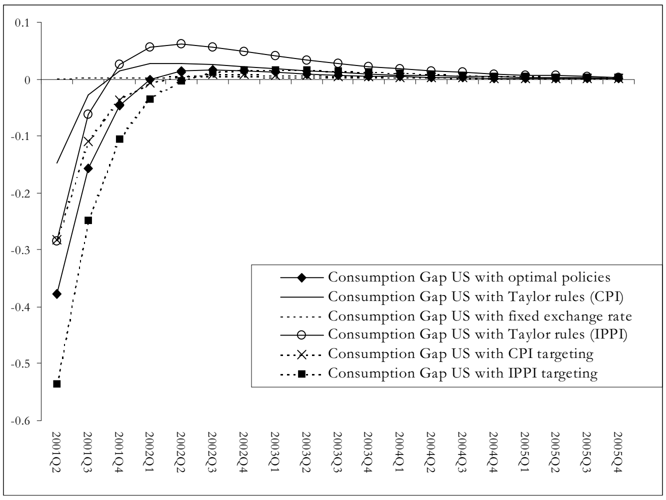
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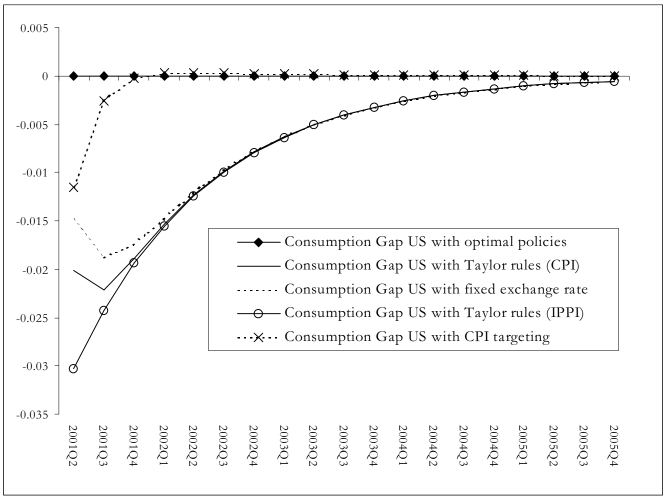
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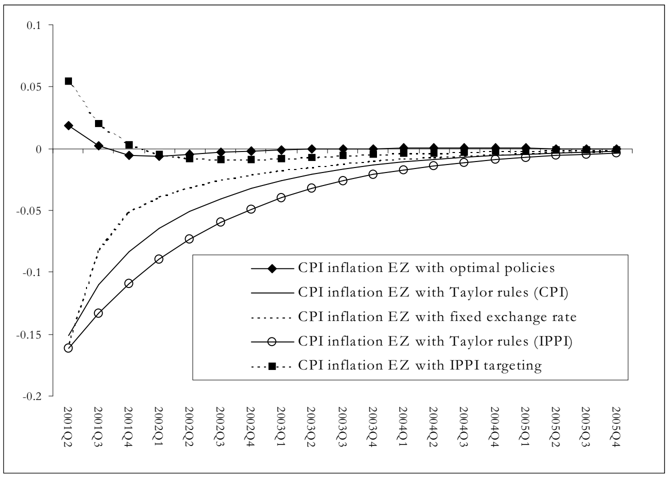
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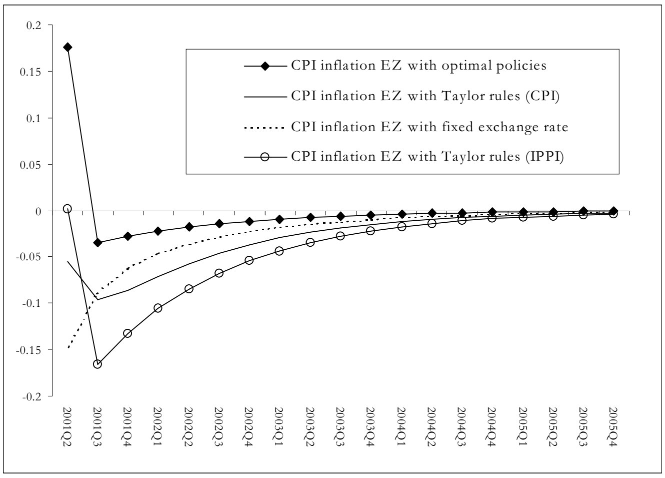
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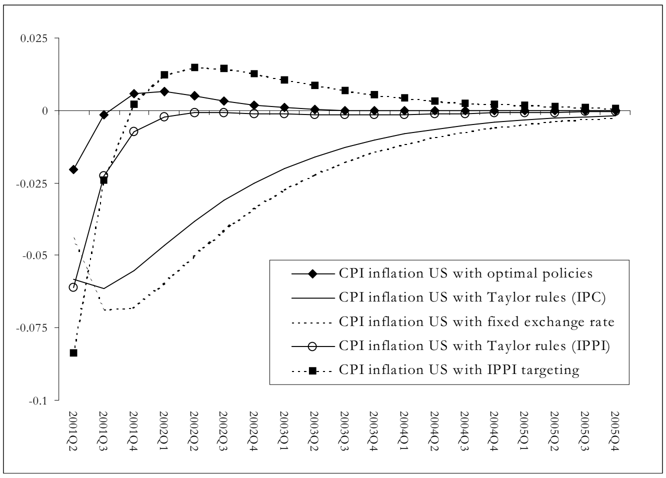
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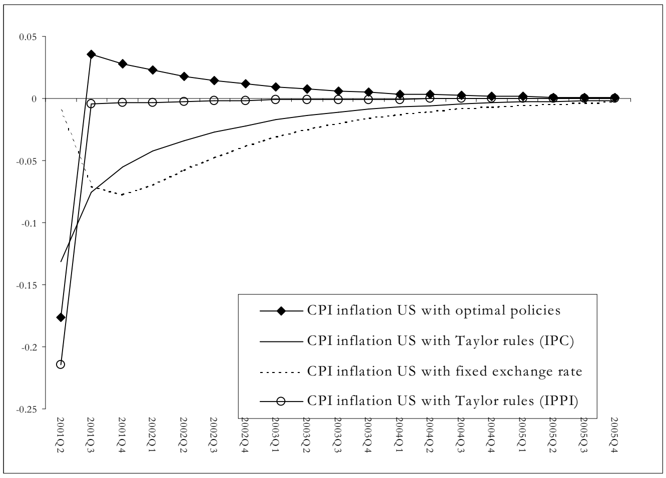
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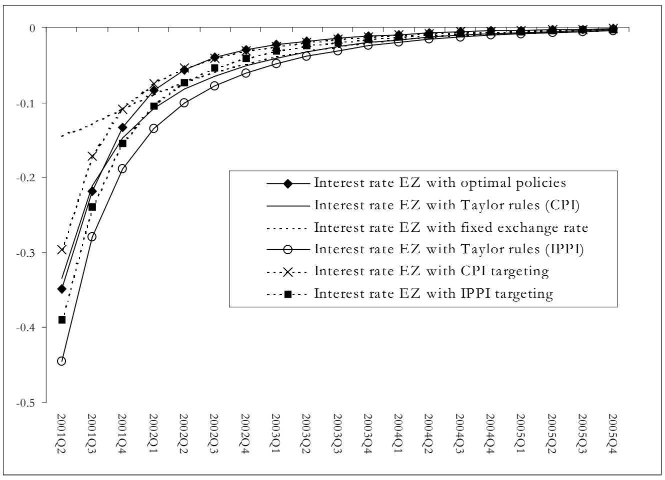
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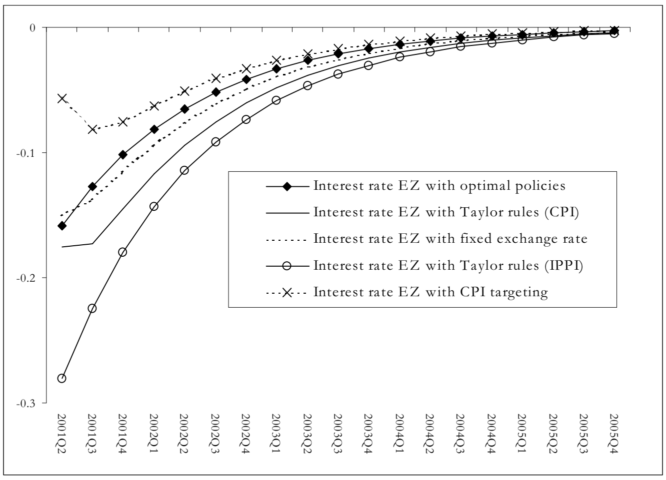
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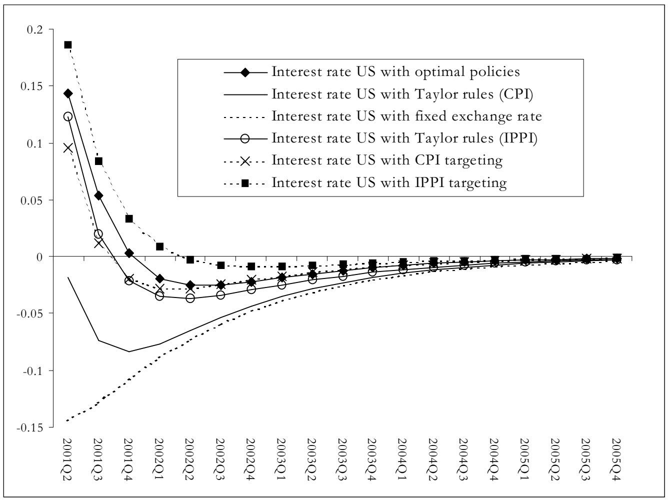
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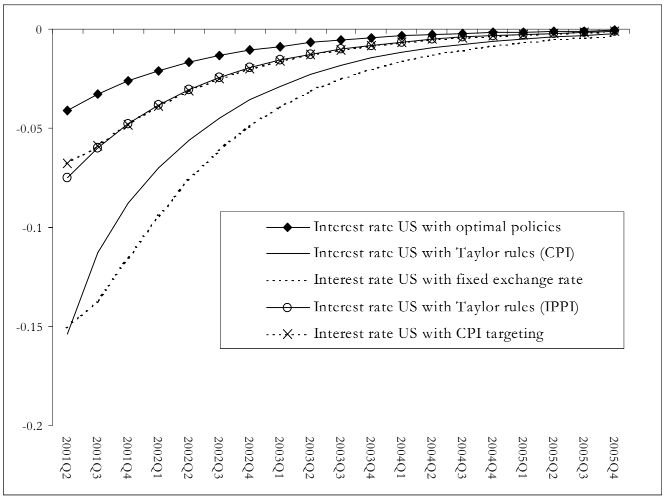
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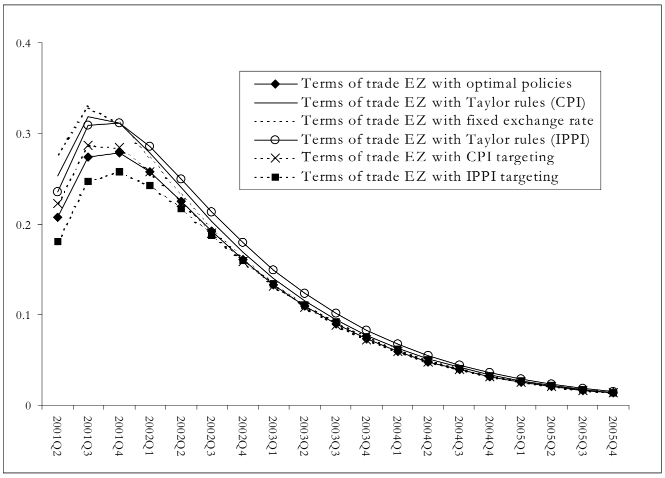
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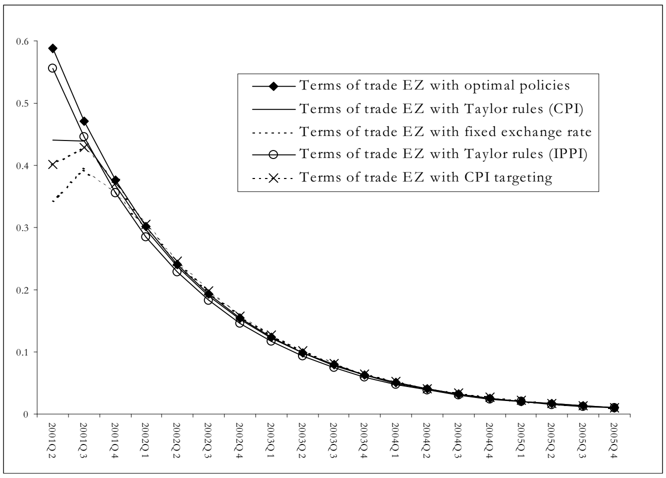
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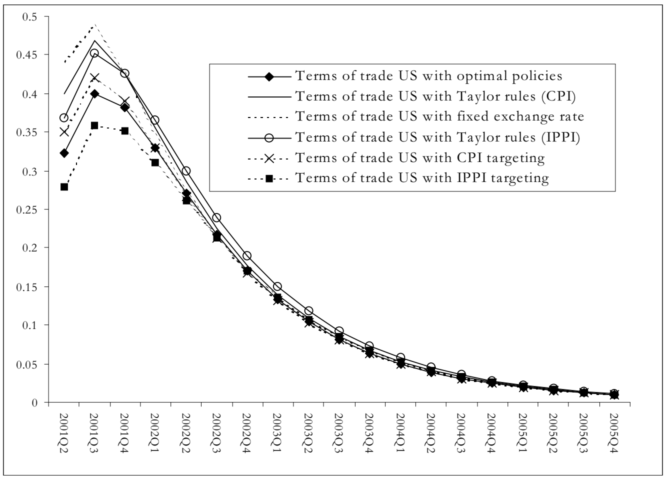
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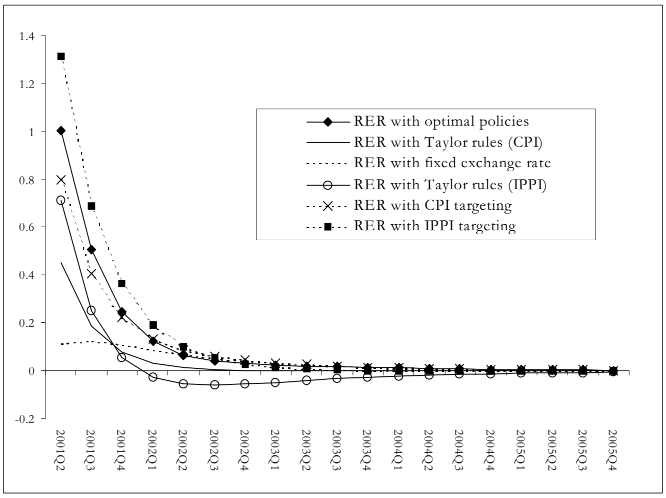
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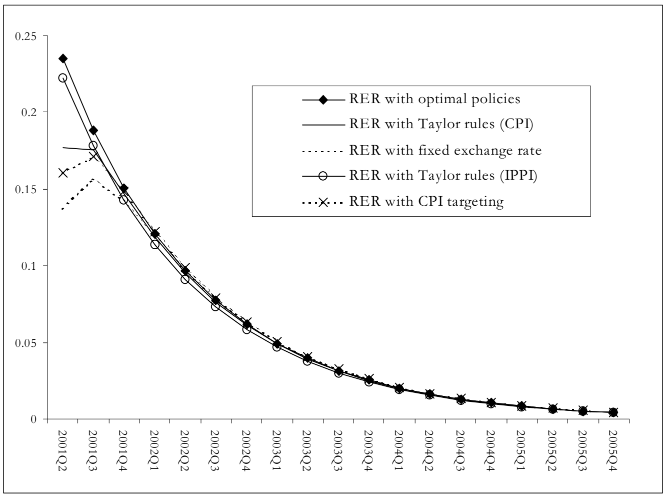
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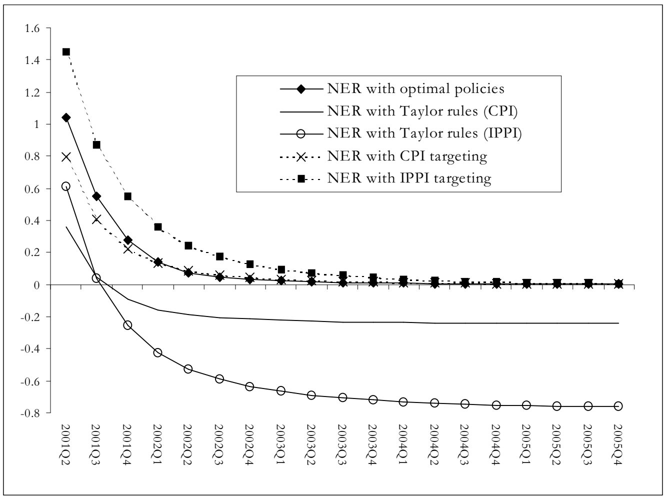

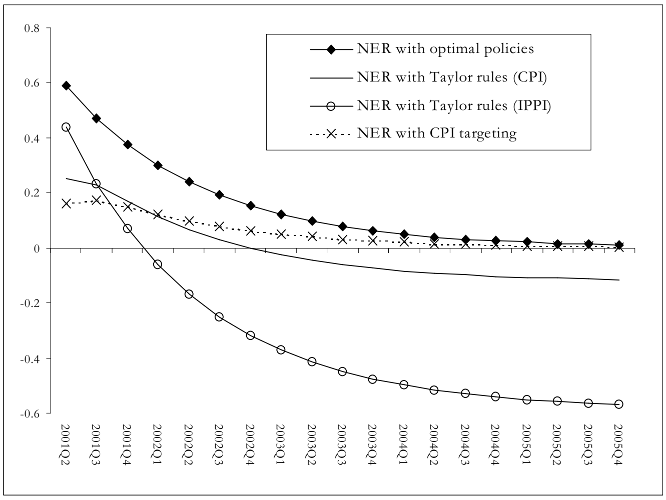

2. Asymmetric "cost-push" shock

In this section, we assume that Euro zone is affected by a "cost-push" shock amounting *ex ante* to 0.25% of the quaterly inflation rate. The persistence coefficient of the shock is fixed at 0.25.

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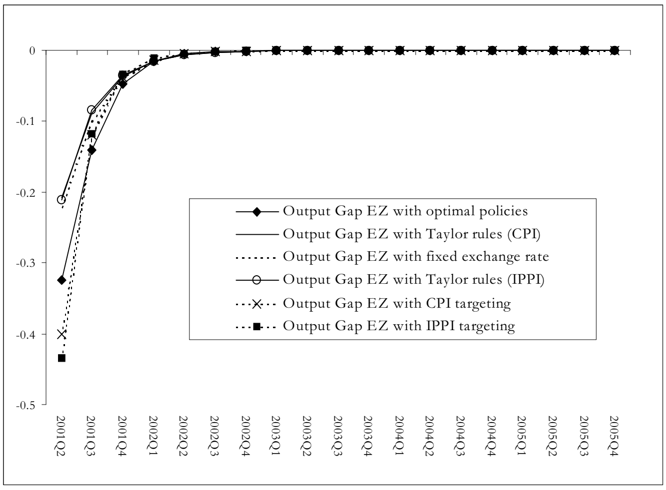

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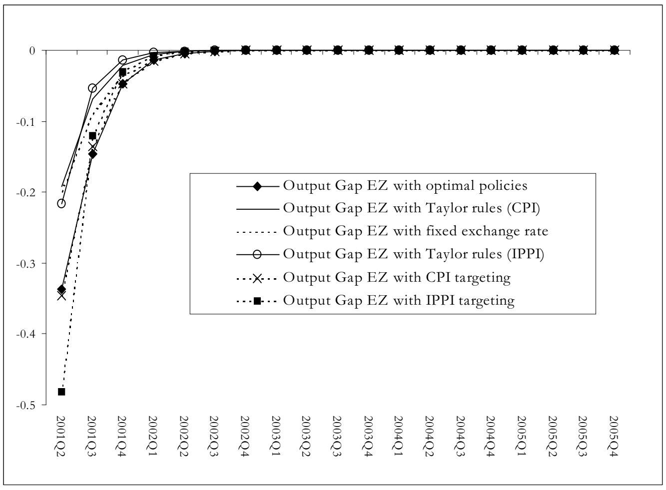

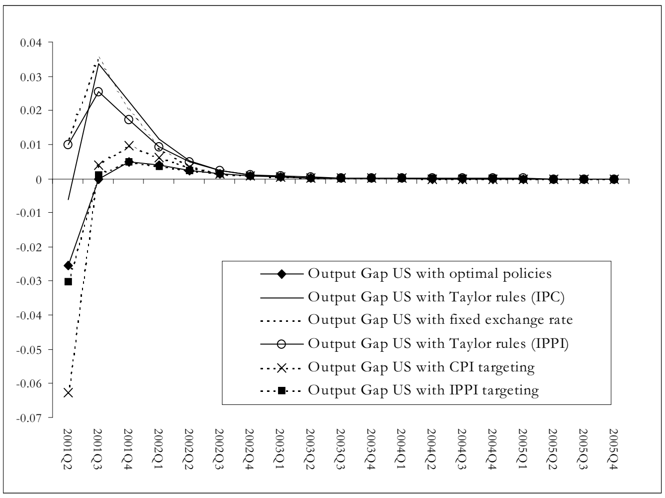

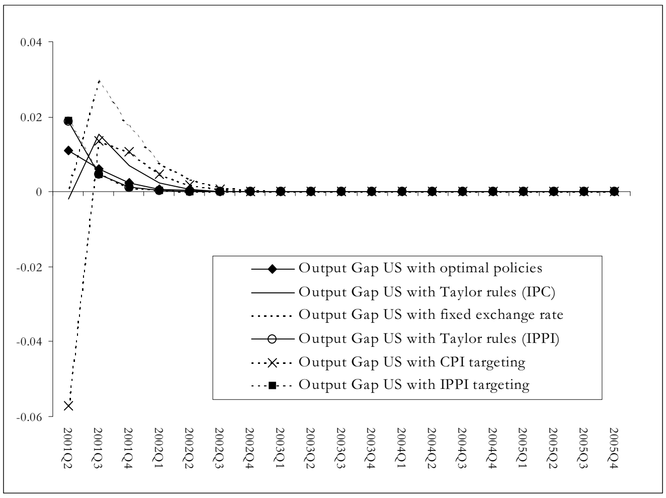

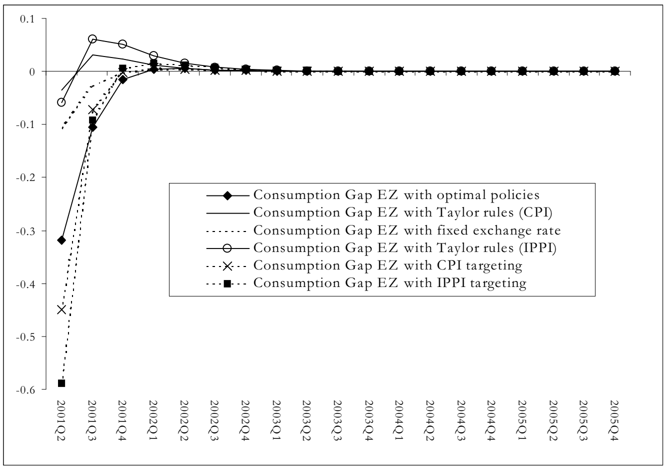
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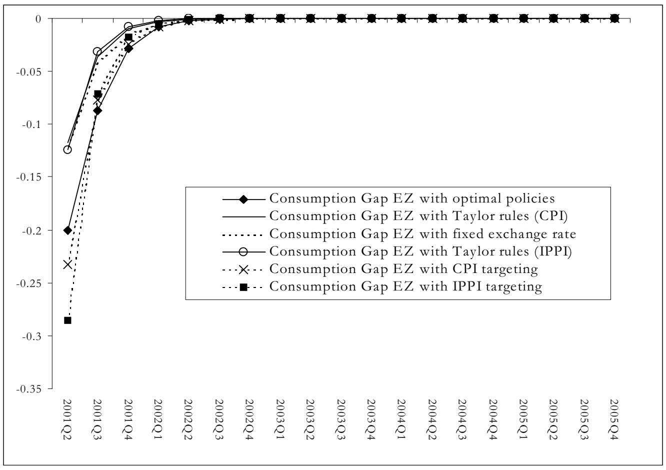
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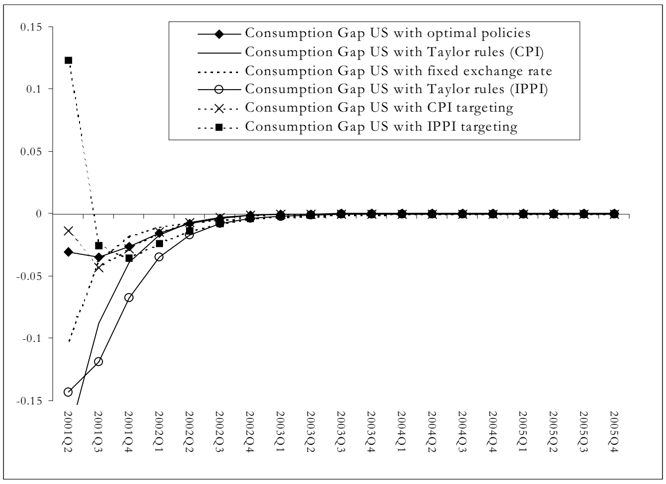
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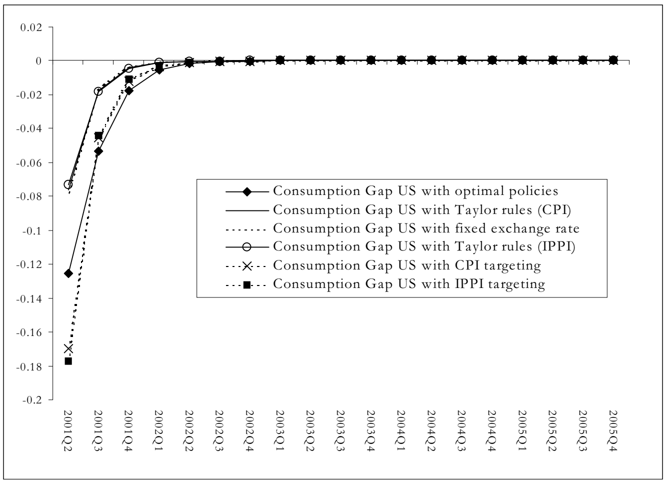
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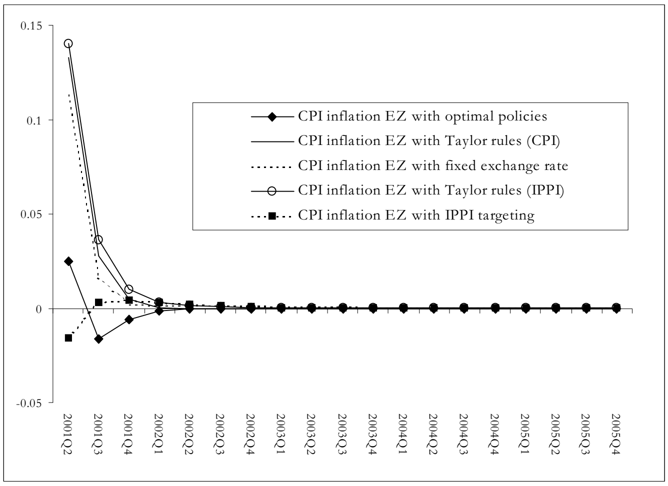
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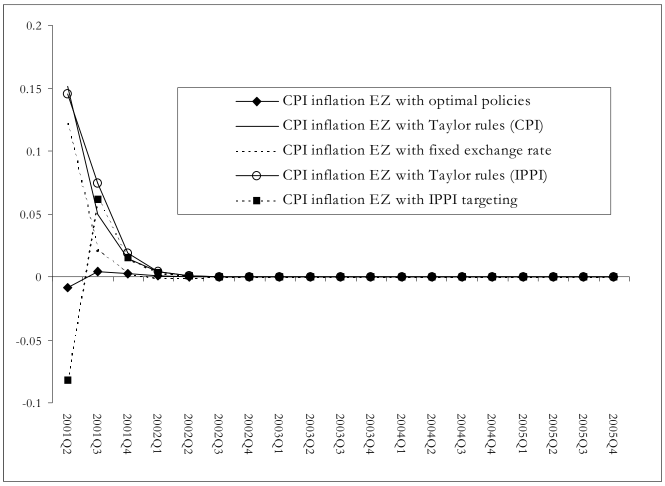
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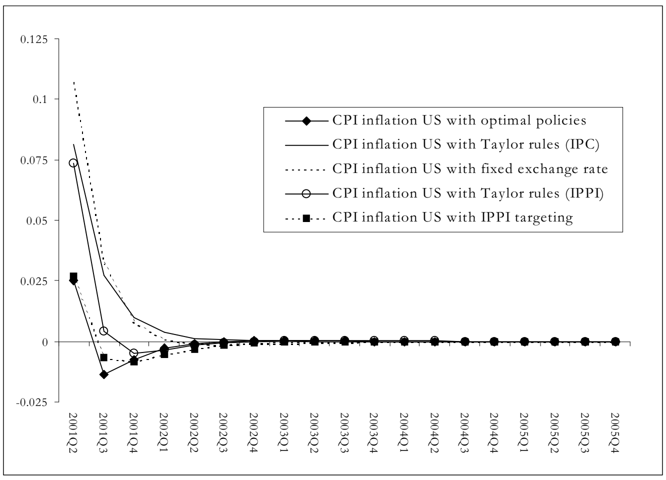
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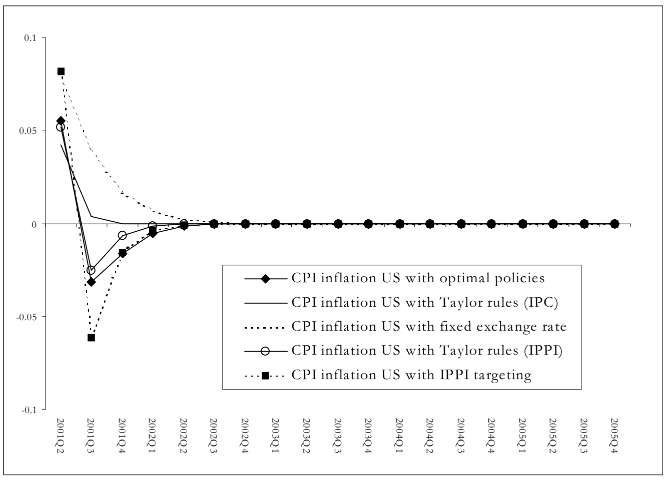
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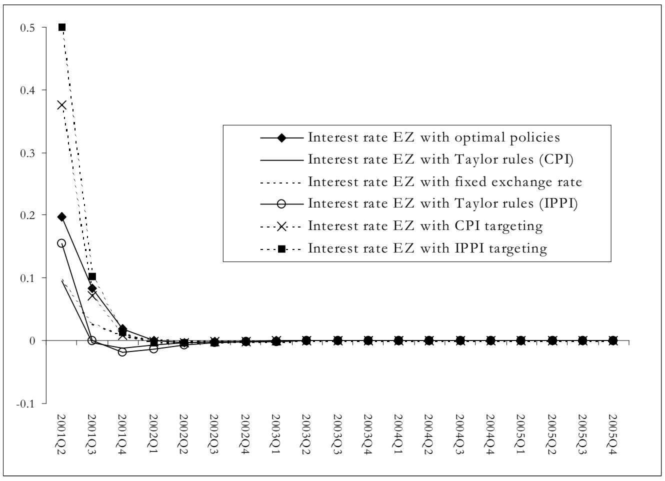
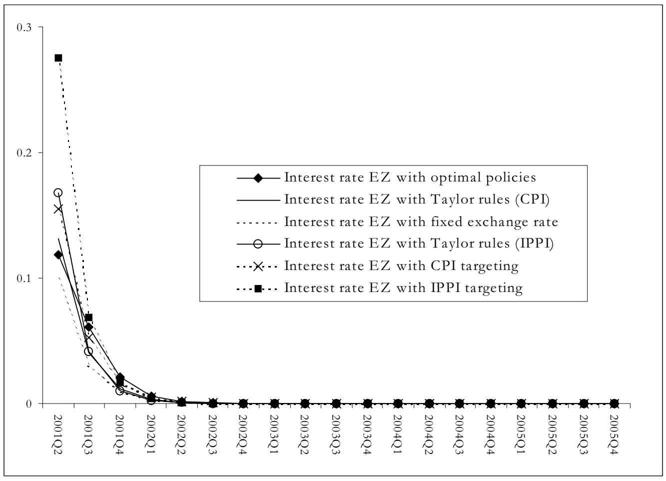
CPI inflation US with Taylor rules (IPPI)
CPI inflation US with IPPI targeting


Interest rate EZ with optimal policies
Interest rate EZ with Taylor rules (CPI)
Interest rate EZ with fixed exchange rate
Interest rate EZ with Taylor rules (IPPI)
Interest rate EZ with CPI targeting
Interest rate EZ with IPPI targeting


Interest rate EZ with optimal policies
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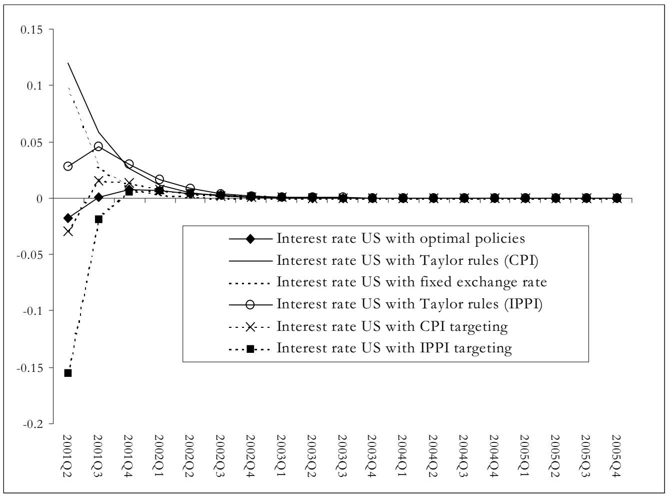
Interest rate EZ with IPPI targeting


Interest rate US with optimal policies
Interest rate US with Taylor rules (CPI)
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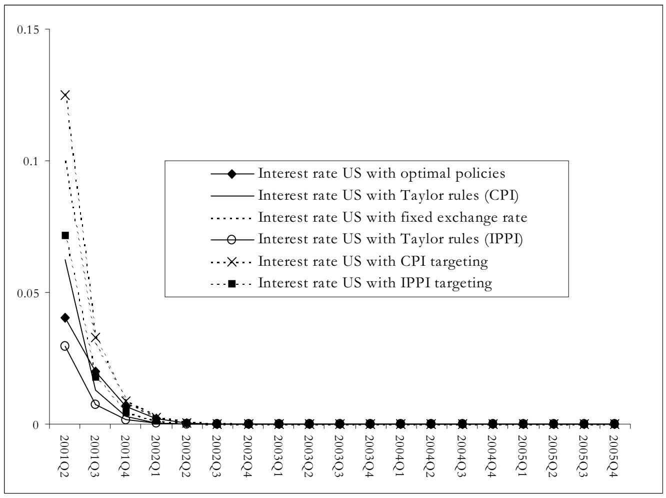
Interest rate US with IPPI targeting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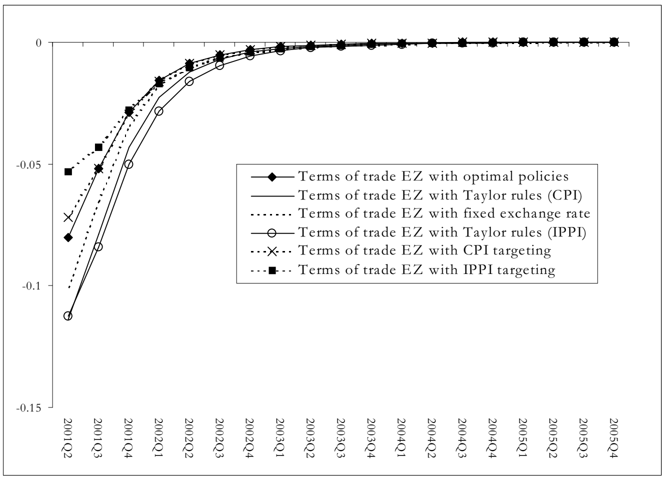
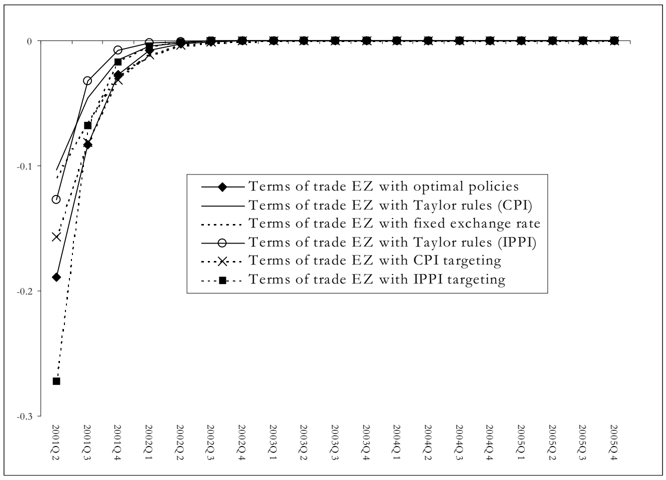

Terms of trade EZ with optimal policies
Terms of trade EZ with Taylor rules (CPI)
Terms of trade EZ with fixed exchange rate
Terms of trade EZ with Taylor rules (IPPI)
Terms of trade EZ with CPI targeting
Terms of trade EZ with IPPI targeting


Terms of trade EZ with optimal policies
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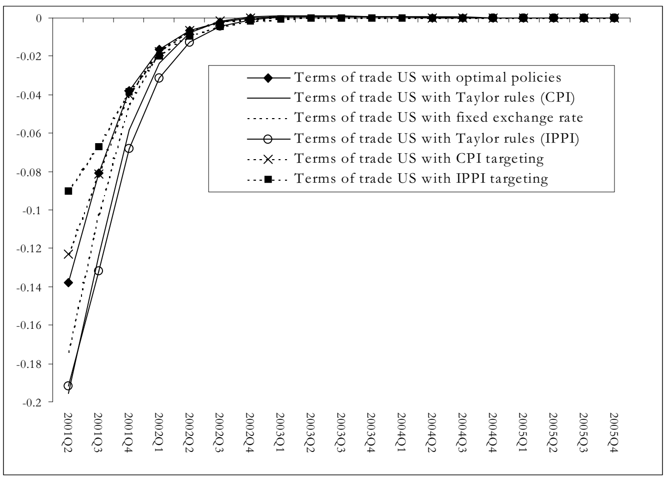
Terms of trade US with optimal policies
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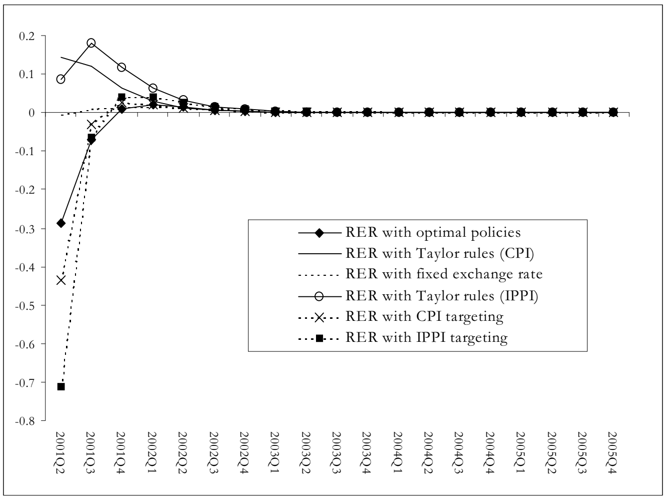

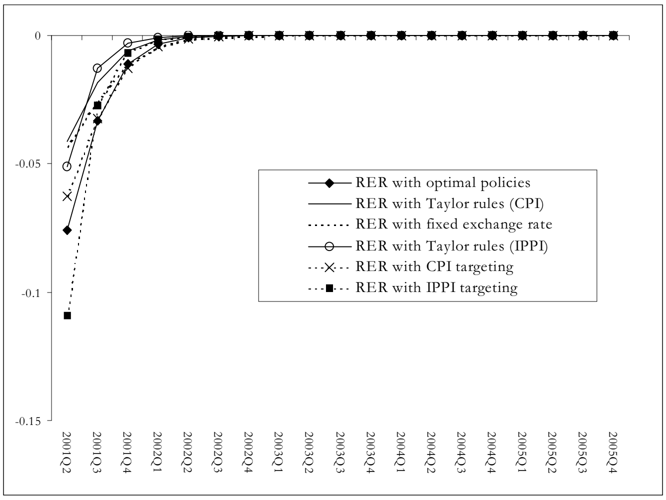

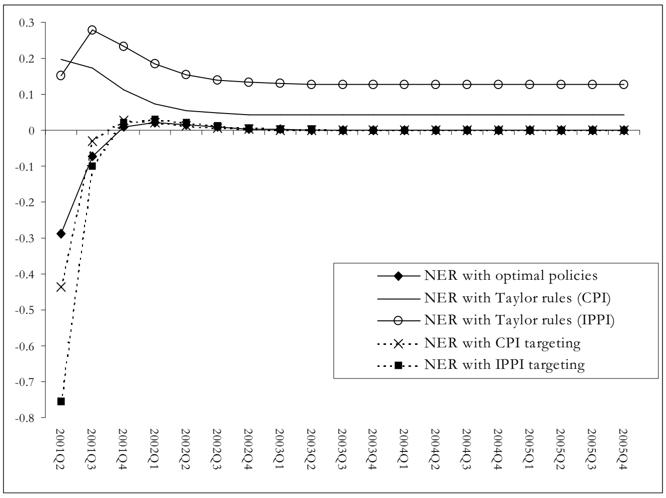

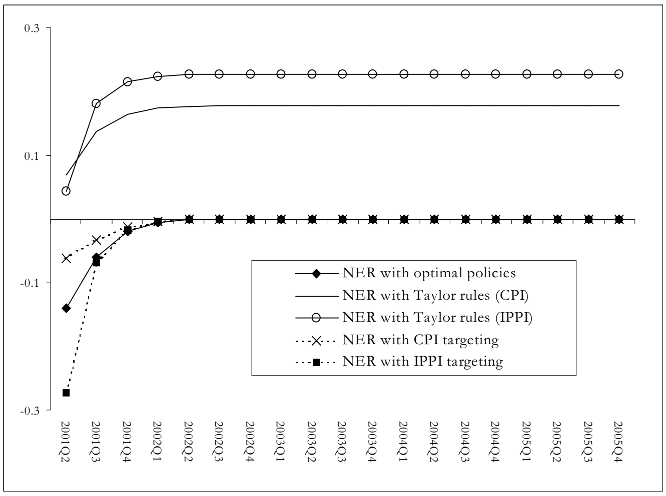

3. Welfare analysis

Table 1: Welfare cost of different policy rules (0/000 shift of steady state consumption)

	"Cost-push" shock				**Technological shock**			
	LCP		**PCP**		**LCP**		**PCP**	
n	0.5	0.7	0.5	0.7	0.5	0.7	0.5	0.7
OPT	0.33	0.30	0.27	0.26	0.72	0.44	0.00	0.00
IPC	0.36	0.34	0.32	0.30	0.72	0.47	0.75	0.44
PQI	0.70	0.38	0.35	0.35	2.02	0.53	0.00	0.00
FIXE	0.80	0.79	0.78	0.81	2.47	2.51	2.64	2.64
TaylIPC	1.58	1.52	1.24	1.24	2.91	3.31	2.30	2.44
TaylPQI	1.52	1.46	1.29	1.29	4.42	4.41	3.06	3.21