Ageing of the labour force: which implications for productivity, training or wages at a macro level?[1]

Population ageing is at the centre of many interrogations. Most of them concern the future financing of pension systems, or other social expenditures going more than proportionally to older age groups, such as health expenditures or expenditures for old age dependency. But part of these interrogations address the consequences of ageing *within* the labour force, which are sometimes presented as being at least as important as consequences of ageing *outside* the labour force. The purpose of this paper is to propose a critical appraisal of this belief. Two main aspects of this question will be examined

The first one is an old concern of populationist writers, who stress the negative impact that population ageing may have for productivity and general economic performance. Simple numerical simulations show that the sign of this impact is uncertain and that, positive or negative, it is probably low in absolute value. This does not mean that there are no justifications to developing training policies aiming at increasing the productivity of older workers. But this policy should better be viewed as a natural inflexion of training patterns in front of a changing demographic structure, rather than a defensive policy aiming at containing dramatic declines of average productivity.

The second question is the question of the implications of labour force ageing for labour costs. This question is to be raised if we admit the possibility of a disconnection between the evolution of productivity and the evolution of average wages over the life cycle, in the spirit of Lazear (1979). Recent estimates by Crépon, Deniau and Perez-Duarte (2001) suggest that such a phenomenon is observable on a panel of French firms. We shall propose a simple application of their results to French demographic perspectives. Here again, the interaction between this effect and the ageing of the labour force remains far from generating a dramatic increase of unit labour costs. This increase may however cumulate with other factors of labour cost increases, in particular those linked with ageing *outside* the labour force, such as pension and health expenditures. Furthermore, at the micro level, this wage-productivity gap should go on generating a pressure to exclusion of older workers from the labour market, and therefore increase difficulties to increase the age at exit from the labour force.

The paper will be organised as follows. Section 1 will recall the basic facts about past and projected ageing of the labour force in France, with a simultaneous look at "natural" ageing, i.e. ageing which would have occurred or should occur in the absence of any changes of actual limits to active life, and effective ageing, i.e. the one which takes place once changes of ages at entry and at exit from the labour force are taken into account. Section 2 will propose simulations of the interaction between these trends and two deliberately contrasted profiles for the age-productivity relationship, showing that the final impact of these interactions falls within a very narrow range. Section 3 will examine the impact of ageing on training policies, in the context of a simple model of optimal accumulation of human capital. Section 4 will propose the final projections of unit labour costs given assumptions on diverging age and productivity profiles over the life cycle and briefly concludes with a discussion of their consequences.

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[1] Didier BLANCHET, INED and INSEE. Paper prepared for the CEPII meeting "Ageing, skills and labour markets", Nantes, 7-8 September 2001. Preliminary version (please do not quote). Part of this paper relies on Blanchet (1992).

1. Past and projected ageing of the labour force.

Figure 1 gives one standard measure of ageing within the population of working age which is the ratio of people aged 50-60 to the whole population of the 20-60 age group. This figure is derived from the most recent national demographic projections (Brutel, 2001). It gives a certain number of interesting results. It confirms that we are currently engaged in a process of "natural" ageing of the labour force, but it also shows that this process is now near its end. If we accept the medium fertility scenario (1.8 children per woman), this share of older workers among potential workers should completely stabilise after 2006, and should even go down slightly after 2020 if we assume a progressive return to replacement fertility (2.1 children per woman). Furthermore, even if this ageing process is significant, it is not unprecedented. It leads to levels of this ratio that are only slightly higher than those experienced at the end of the 1960s.

It is interesting to trace more closely the mechanisms behind these past and future trends in population ageing. Until the end of the 1960s, ageing of the labour force is due to the progressive entry, in the 20-60 age group, of the small cohorts born until the mid 40s. After 1965, they are followed by the first baby-boom cohorts, which induce a rejuvenating process, which has been temporarily amplified by the arrival, in the 50-60 year group, of the war cohorts born between 1915 and 1919. These cohorts depressed the relative weight of the 50-60 age group between years 1965 (when cohort 1915 entered this age group) and 1979 (when cohort 1919 left this group). Without this demographic accident, the rejuvenating of the labour force between 1965 and the mid 1990s, induced by baby-boomers, would have followed a more continuous and regular path, until 1996. After this year, this trend reversed, because the first baby-boom cohorts started joining, in turn, the 50-60 age group, and because of the simultaneous entry, at the bottom of the 20-60 age group of smaller cohorts born after 1975 (it is around 1965 that fertility started to decline in France but births actually declined later, during the mid 1970s).

This process is the one that is still currently at work. It will last until the first baby-boom cohorts will finally reach age 60, i.e. around 2006. After this period, these baby-boom cohorts will go on feeding the general ageing process, by increasing the share of people over 60, but ageing *within* the labour force will, on its side, reach some kind of steady state. At this steady state, the share of the 50-60 age group among the 20-60 age group should stabilise at about 30%, which can be readily compared to the share of 25% that we should normally have had with a purely uniform age distribution between 20 and 60 (and to which we would approximately return if fertility were to stabilise at replacement level).

One problem raised by this first index of ageing is its sensitivity to the conventional threshold of 50 used to isolate the subgroup of older workers. Using a threshold also has the drawback of leading to sharp evolutions in case of large breaks in the age pyramid. This is legitimate if this threshold has an institutional significance, such as the threshold of 60 for pensions. But the threshold of 50 does not have any equivalent meaning (it only had one, occasionally, for some very specific schemes of early retirement). A more appropriate measure of labour force ageing is therefore provided by the mean age of the labour force or of the working age group. The mean age of the 20-60 age group is given on figure 2. We naturally find the same qualitative evolutions than on figure 1, but smoother and with a slightly different timing.

Figure 1 : Share of people over 50 in the 20-60 age group

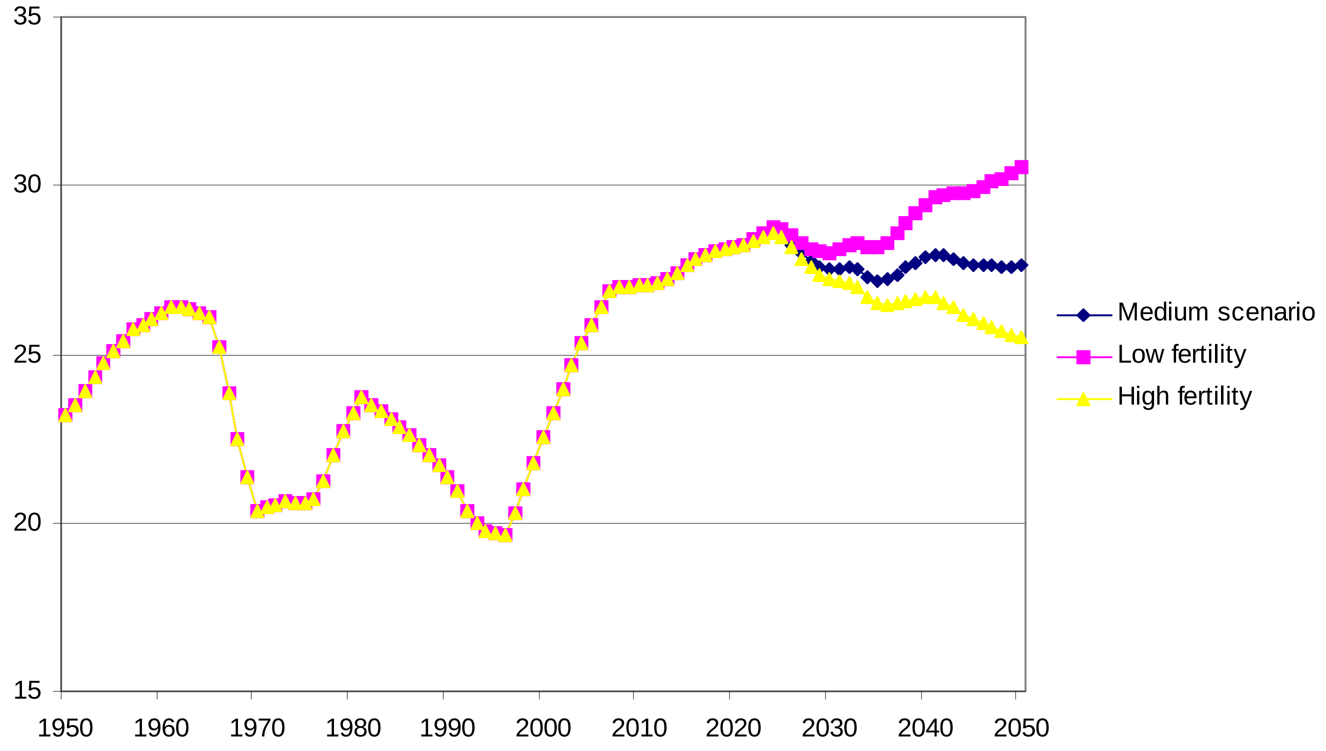
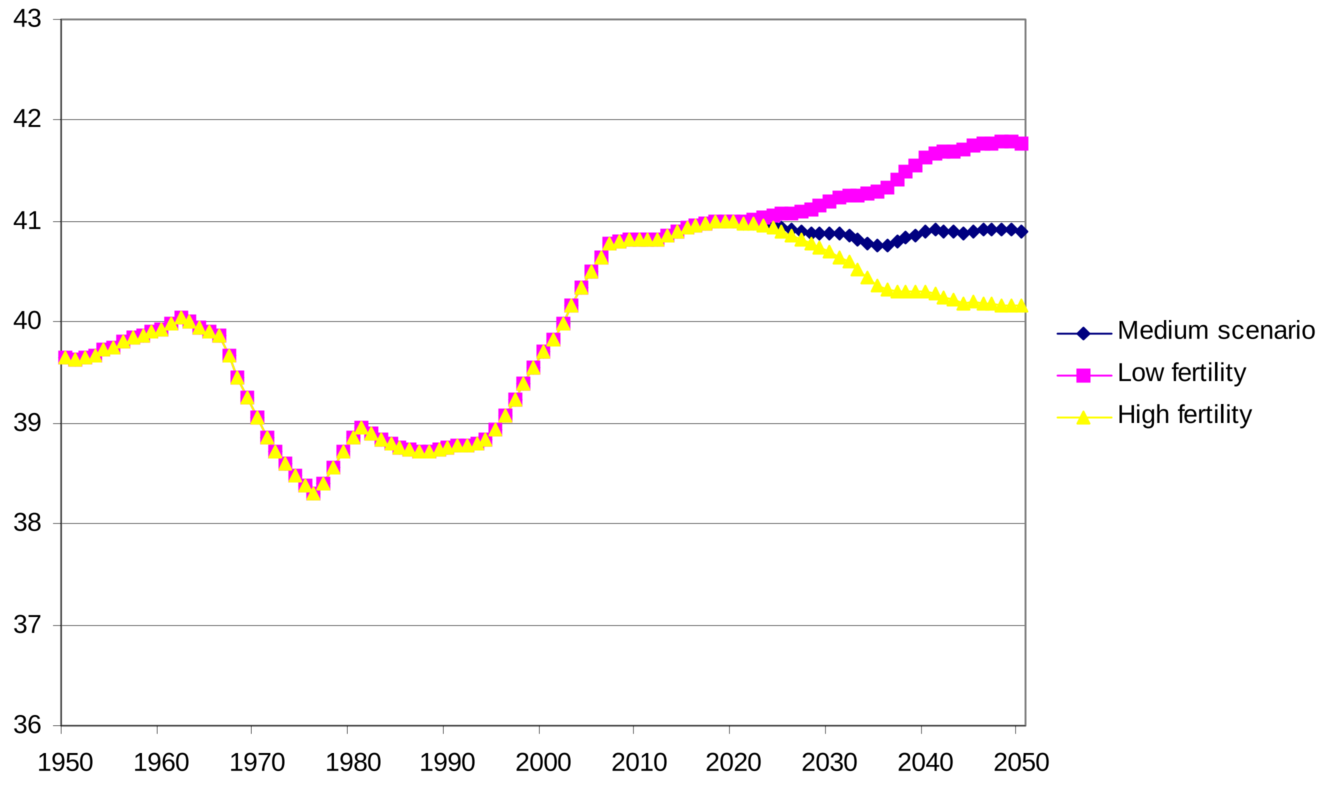


Figure 2 : Mean age of the 20-60 age group



In particular the rejuvenating process induced by the arrival of successive baby-boom cohorts on the labour market stops in 1985, when the oldest of these cohorts enter the second half of their active lives, instead of lasting until these cohorts reach age 50. Most important, the global changes in the age structure illustrated by figure 1, which could look important, appear much more modest when translated in terms of changes of the mean age of the potential labour force, which varies in a bracket comprised between 38,5 and 41 years.

Are these pictures altered if we shift from measures of ageing within the potential labour force to measures of ageing in the *actual* labour force? The first measures only consider the impact of pure demographic components, what we called “natural” ageing in the introduction[2]. Measures of ageing within the actual labour force will also include the impact of spontaneous or induced changes in labour force participation which, if differentiated by age, will lead to a distinct pattern of ageing. One particular question is to know whether policies aiming at early withdrawal from the labour force, since the mid seventies, actually played the role of mitigating the process of natural ageing.

Figures 3 and 4 show that this is only partially the case. These figures give the two previous measures of ageing within the 20-60 age group, restricted to the period 1975-2025 and to the medium fertility scenario for the period after 2000, and confront these measures to the ones obtained on actual labour force, given observed LFP rates for the period 1975-2000, and assuming constant LFP rates after 2000.

It is essentially around 1980 that natural and actual ageing have gone in opposite directions. Until the end of the 70s, actual age at exit from the labour force remained significantly above 60, but this gap progressively eroded: this was due first to the development of early retirement schemes below 65 (which still was the normal age at retirement), then to the introduction of normal retirement at age 60, in 1983, followed during a few years by further developments of early retirement schemes below 60, more or less stabilised since the mid 90s. Other factors, such as the tendency to a later entry into the labour force also played a role in these evolutions. It is well-known that the French specificity in terms of LFP rates is not only the low age at exit from the labour force, but more generally a phenomenon of concentration of the active life-cycle over median ages. Delayed entries partly compensated the artificial rejuvenating due to earlier exits from the labour force. Their impact is higher on the mean age than on the share of people over 50, and this probably explains that this mean age, in the actual labour force, started re-increasing about 10 years earlier than in the full 20-60 age bracket.

Concerning future trends, ageing in the “actual” LF and in the 20-60 age group are, by construction, quite similar, the basic hypothesis being one of stability of future LFP rates. Some variants of this assumption concerning LFP rates are currently under preparation, to be used for the new labour force projections to be issued by INSEE in 2002. Among these variants, the ones that should have the more impact on “actual” ageing are the ones concerning retirement age. This is the reason why, on figures 5 and 6, we propose variants with a large change of this retirement age, supposed to increase by 5 years between 2005 and 2025.

[2] In a sense of the word “natural” larger than the one usually considered by demographers, since it also includes the impact of migration flows.

Figure 3 : Share of people over 50 in the actual labour force and the 20-60 age group

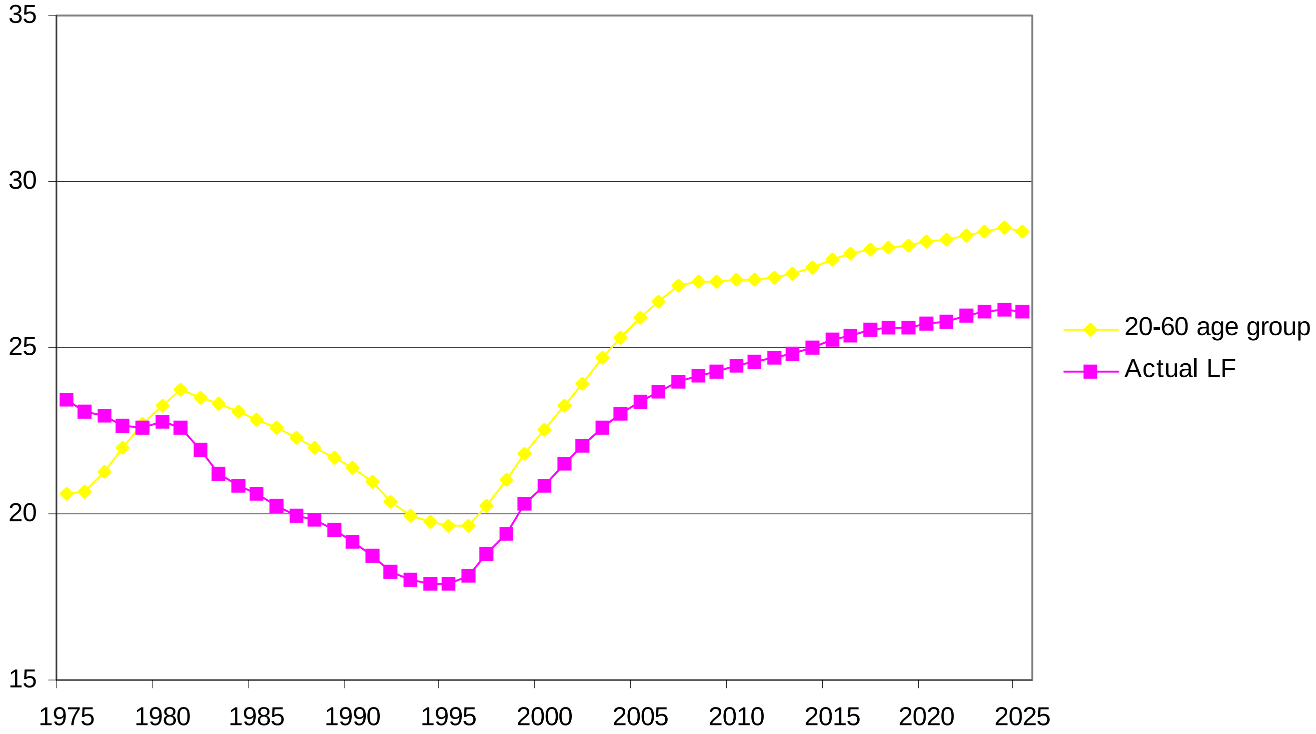


Figure 4 : Mean ages of the actual labour force and the 20-60 age group

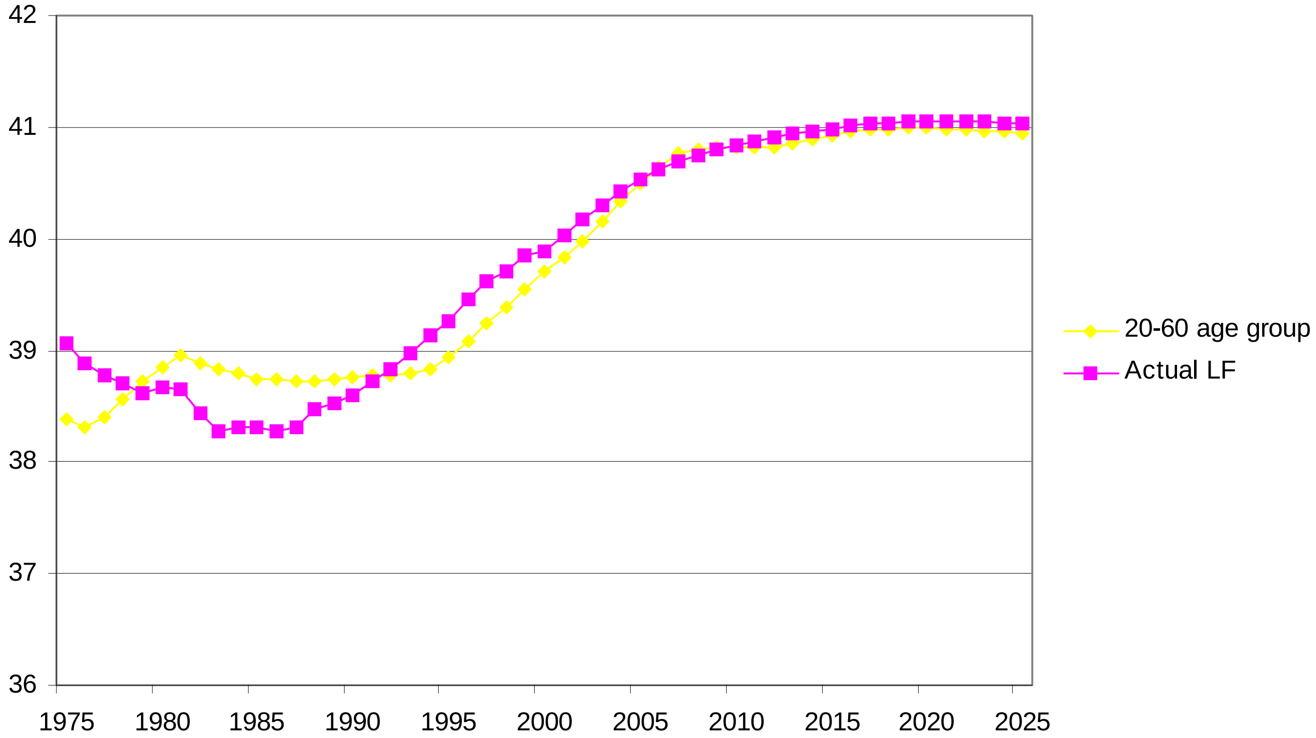


Figure 5 : Share of people over 50 in labour force : fixed LF participation and delayed retirement age

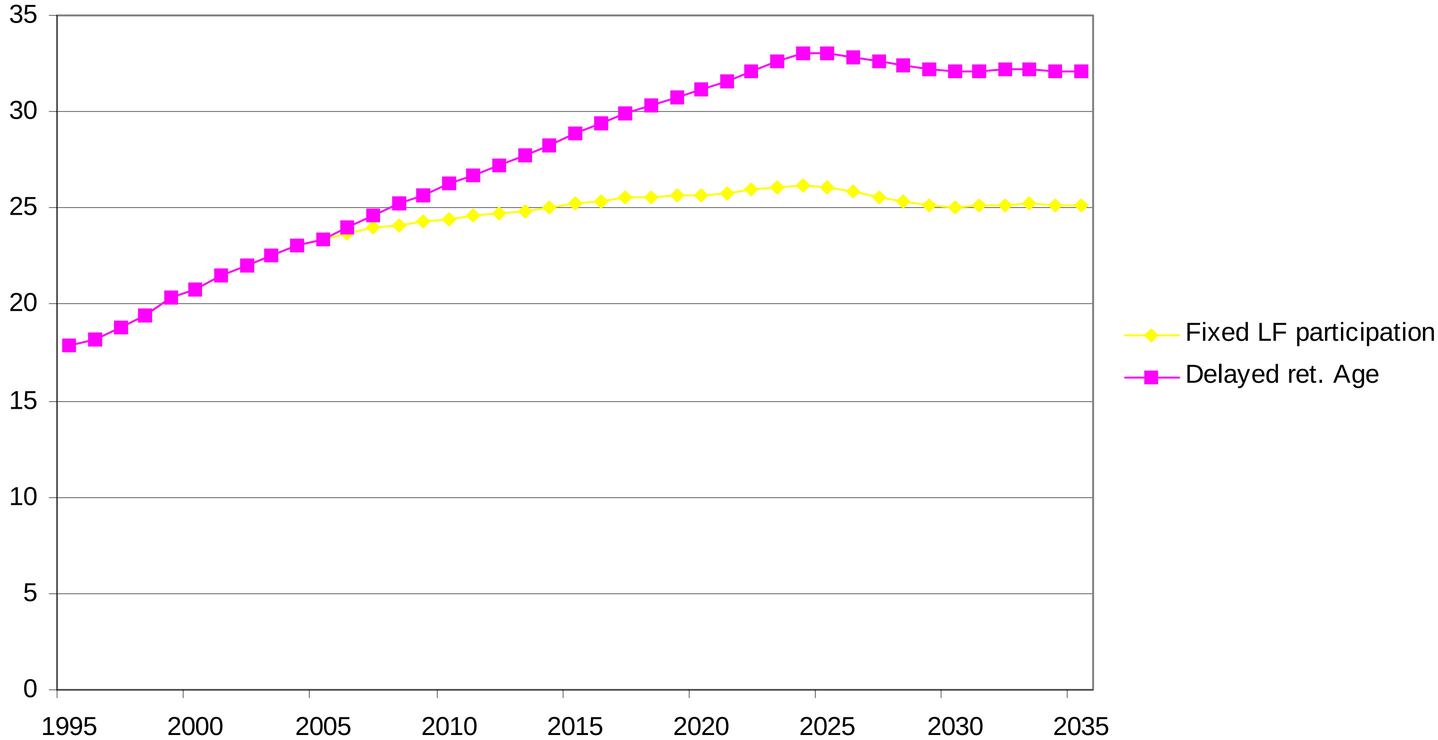
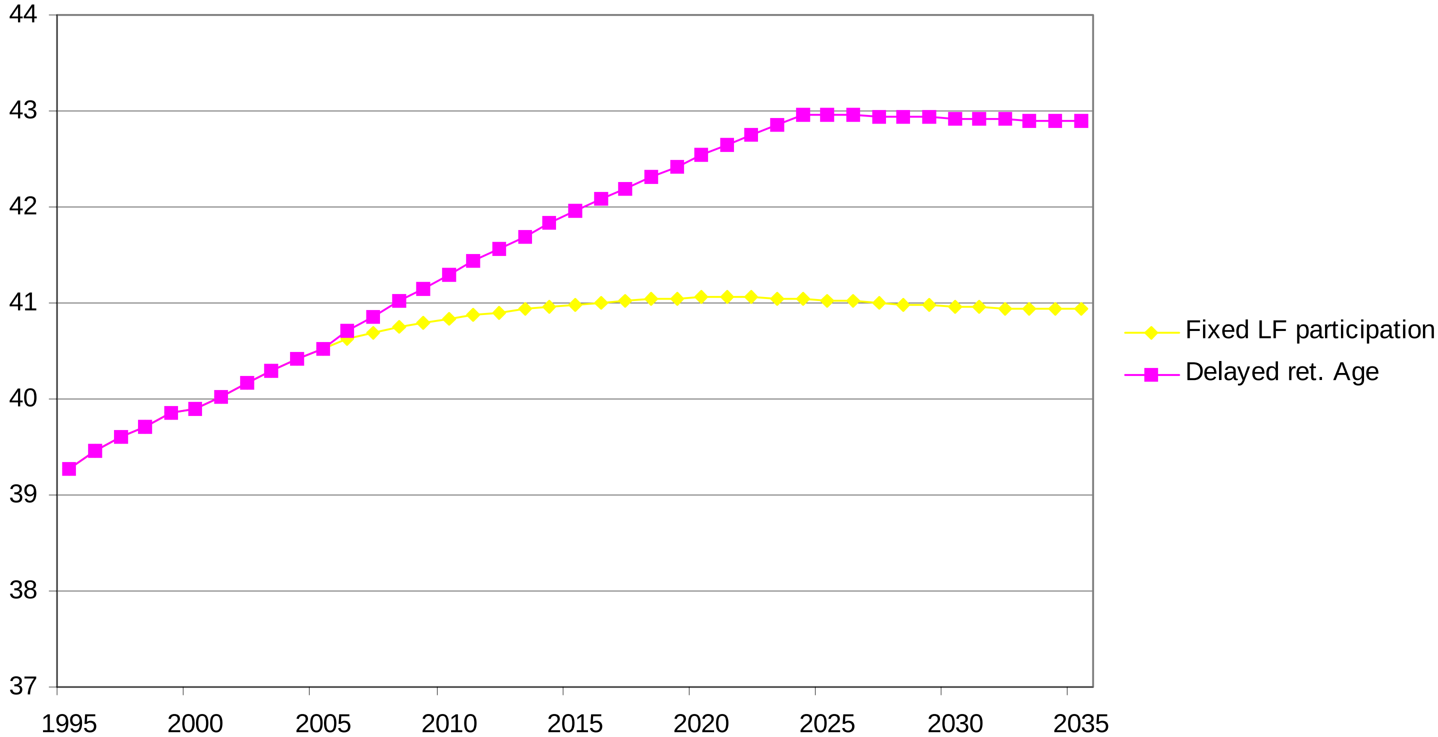


Figure 6 : Mean age of the labour force: fixed LF participation and delayed retirement age



The impact is the one that could be expected. The share of people aged more than 50 in the labour force stabilises about 7,5 percentage points above its steady state level with no change in the retirement age. The mean age of the labour force stabilises about 2 years higher than with a fixed retirement age.

In sum, the conclusions of this first section are the following: purely demographic factors explain an ageing of the labour force which should not extend beyond 2006, and lead to indicators of ageing only slightly higher than the maximum values registered over the past 50 years. Ageing would be more pronounced is we add to this "natural" ageing a process of "artificial" ageing due to a policy of increasing retirement age.

2. Ageing and average productivity

What can be the consequences of these trends for average productivity? Let us start with very simple assumptions of a fixed age-productivity relationship. What we know about this age-productivity relationship is generally indirect and considered as uncertain. For this reason, we are going to consider arbitrary theoretical profiles, deliberately very contrasted. The purpose is not to produce accurate predictions, but to produce tests of sensitivity.

Two profiles will be used, given on figure 7. The first one is purely artificial. It assumes a very strong negative transversal impact of age on individual productivity, which could be either the consequence of a real productivity decline with age, or the consequence of a vintage

Figure 7 : Two arbitrary profiles for the age-productivity relationship

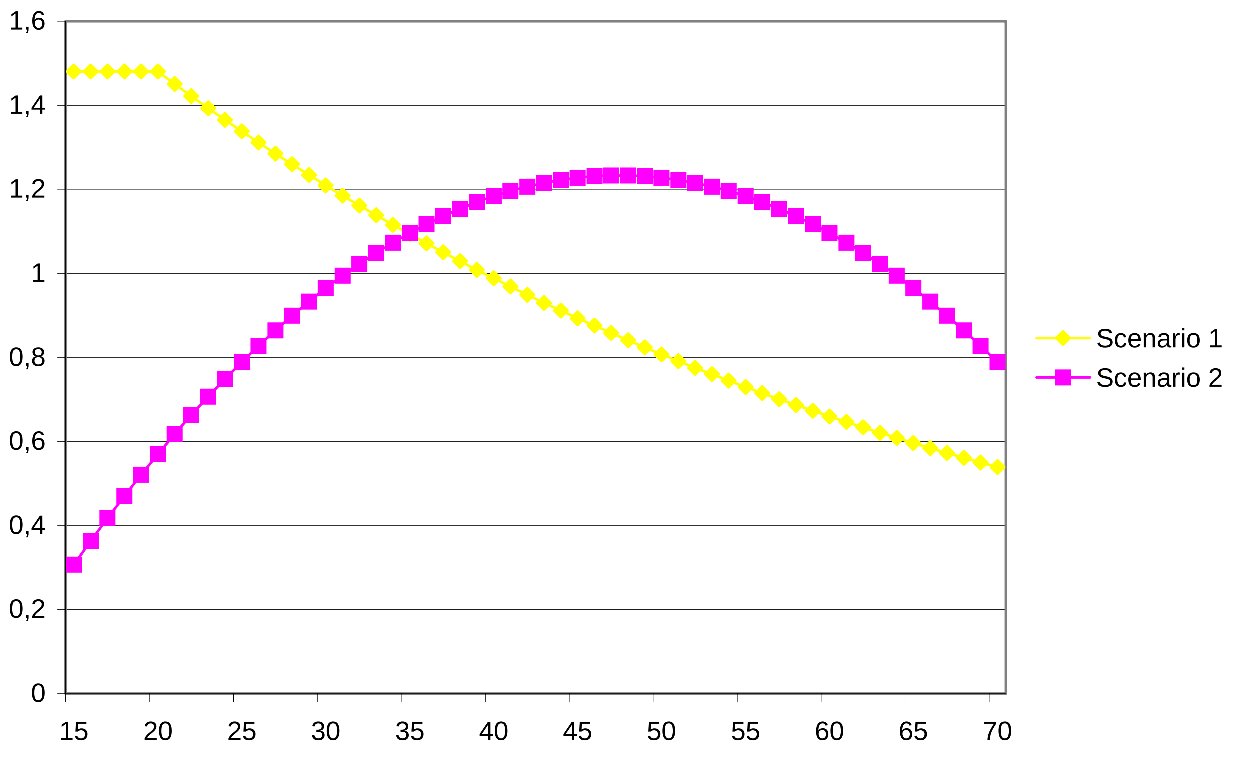


Figure 8 : Simulated average productivity, scenario 1 (productivity declining with age), base 100 in 2000

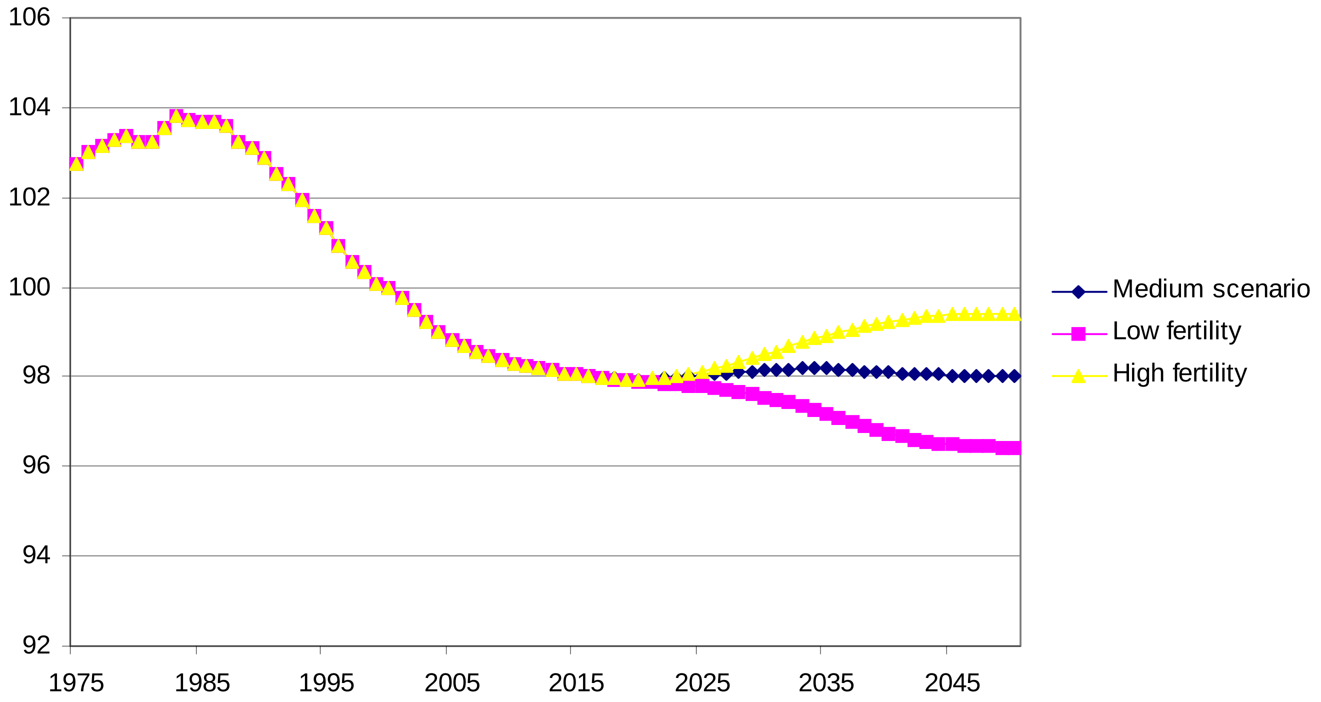
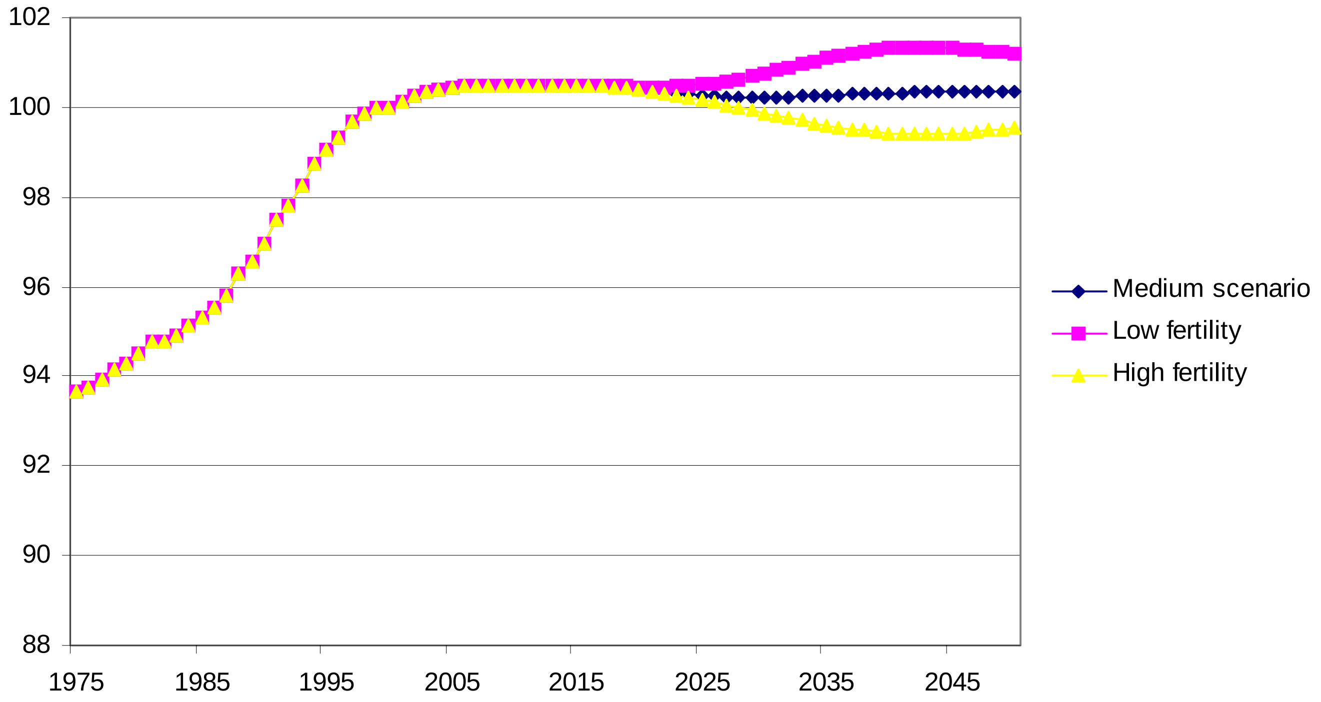


Figure 9 : Simulated average productivity, scenario 2 (hump-shaped productivity), base 100 in 2000



effect, i.e. the fact that younger generations, trained more recently, are able to use more productive techniques. The purpose of this first assumption is definitely not to produce predictions, but to show that, even under a very strong assumption, the impact of ageing on productivity remains surprisingly low. This is the reason why we chose a rate of productivity decline with age deliberately high, at 2% per year of age, past age 20, corresponding either to this rate of productivity decline with age, or to this rate of exogenous technical progress embodied in successive cohorts of workers. It can also correspond to a mix of these two hypotheses, for instance a 1% decline of productivity per year with age, combined with a 1% rate of exogenous embodied technical progress.

The second profile is a hump-shaped profile, typical of a model of human capital accumulation, where productivity is first an increasing function of age, due either to on-the-job training or learning by doing, and declines only in the later stages of the active life-cycle.

The results of interacting these age profiles with projected demographic structures and age-specific LFP rates, under the three fertility scenarios already used for figures 1 and 2, are given on figures 8 and 9. In both cases, the profiles, normalised to 100 in 2000, present only the pure effects of demographic ageing on productivity, and do not take into account any other factor likely to increase productivity. In particular, if the negative relationship between productivity and age postulated by the first profile was to include the impact of embodied technical progress at a rate of, say, x%, a mechanical result should be a steady state rate of average productivity growth of x%, to be added to the simulated profile, as should be added the impact on average productivity of any trend of exogenous disembodied technical progress.

Profiles shown on figures 8 and 9 are as expected. Under the first assumption, average productivity moves in the opposite direction to the mean ages of the 20-60 age group or of the labour force that were given on figures 2 or 4. Productivity, all else equal, would have peaked at an index level of 104 between 1985 and 1995, and the process of global labour force ageing between 1995 and 2005 should lead us, after 2006, to a steady state index level of 98. It is only after 2020 that other fertility scenarios could potentially lead to new divergences, with new steady state levels ranging from 97 to 99 between the low and replacement fertility assumptions.

With the second productivity assumption, evolutions are roughly the opposite, with the exception of the period 1975-1985 where evolutions are of the same sign: during this period, demographic evolutions are favourable simultaneously to younger groups and groups of median age. It is the first of these two trends that increases productivity in the first scenario, and the second one that has the same effect in the case of hump-shaped productivity. On the whole, in this second scenario the movement of average productivity is monotonically increasing from 1975 to 2005, with an index value going from 94 to a little more than 100, which is the steady state level for the fertility level of 1.8. Other demographic scenarios lead, once again to temporarily diverging evolutions with new steady state levels ranging between 99 (replacement level) and 101 (with a fertility of 1.5).

These simulations raise two questions. The first one is to know which of the two scenarios is definitely the closest -or the least distant- to the actual age-productivity relationship. The second is to understand why, whatever their sign, these impacts of ageing on average productivity remain so moderate and whether this result is robust to other specifications of interactions between age structure and productivity of the various age groups.

Concerning the first question, we have mentioned the lack of empirical evidence concerning productivity declines before later ages. The reader is referred to OECD (1998) and Jolivet (1999) for recent reviews. Recent estimates by Lindh and Malmberg (1999) offer some support to a positive effect of age by estimating a standard growth model with age structure effects on a sample of OECD countries since 1950. The contribution of labour force ageing to total factor productivity growth appears to be positive until 1970 in these countries, representing about 0.3 points of yearly productivity growth. After 1970, it is the rejuvenation of the labour force that contributes, this time negatively, to productivity growth, with an opposite contribution of –0.3 point per year. It must be noted that this conclusion is not the result of unduly attributing to demographic trends the general economic slowdown after 1975, since it is robust to the introduction of period dummies. This result actually means that countries that, over this period, experienced the highest level of labour force rejuvenating were also the ones, all else equal, with the poorest economic performances.

Some more recent estimates, over a panel of French firms, by Crépon, Deniau and Perez-Duarte (2001), to which we shall come back later, rather conclude to relatively flat productivity profiles in manufacturing for the four age groups of workers under 25, between 25 and 34, between 35 and 49 and over 50 years, except for highly skilled ones for whom productivity remains strongly increasing with age during the first half of the active life-cycle. In the service sector, however, one may note patterns closer to the assumption of a productivity which declines with age, i.e. offering some moderate support to our scenario 1, although the results obtained for this service sector are considered, by the authors, as more fragile than those obtained for manufacturing.

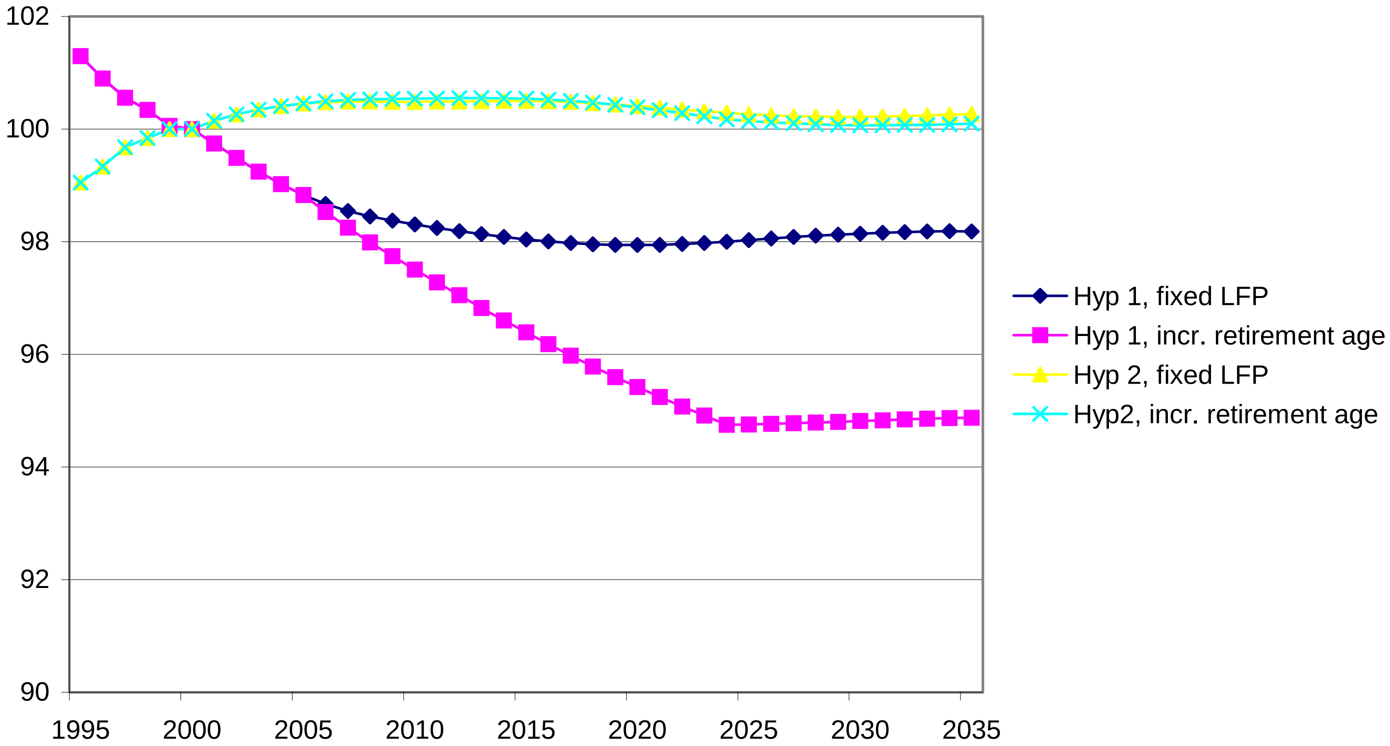
Work clearly remains do be done to clarify these individual age effects, which are interesting for themselves, but one conclusion will anyway remain, which is that, whatever their sign, these effects are not likely to legitimate the fear of very strong adverse effects of future labour force ageing on average economic performance. Why are these effects so weak and is this result robust?

If we stay within the context of a simple linear aggregation of age-specific productivity coefficients, we can link this weakness to some very general results of stable population theory which give simple rules of thumb for assessing the conditions under which a given aggregate will, or will not, be strongly sensitive to demographic change (Blanchet, 1994). In the case of changes of age structure linked to changes in the population growth rate, which is the relevant case concerning the labour force[3], the condition for a (rather) strong effect of population deceleration or acceleration is to have a variable of interest with an age distribution strongly shifted either toward very low or very high ages. This is the case for retirement for which the mean age is about 70, i.e. 30 years higher than the global mean age of the population. But, in the case of the two age distributions used in simulations and despite their asymmetry, the same mean ages remain much closer to the global mean age (or the mean age of total labour force which is roughly the same), respectively 37.5 and 41.3 years, which actually leads to the low sensitivities that we have found.

The fact that these low sensitivities may remain paradoxical is the result of a classical confusion between consequences of ageing at the micro level and at a macro level: the point is that the ageing process at the individual level and at the collective level and the individual

[3] Ageing due to decreasing mortality is an important phenomenon too, and probably the dominant one for pension problems, but only at later ages: its impact on the labour force is almost non significant.

Figure 10 : Average simulated productivities (medium demographic scenario and increasing age at retirement after 2000)



level have orders of magnitudes which are completely different: one individual gains one more year of age every year, while a group (except if it is a completely closed group), ages every year by at most one tenth of a year, as was shown on figures 1 or 3, and does so only for limited periods of time, since stabilisation of the demographic structure always lead to stabilisation of any age-related aggregate.

Appendix 1 gives the details of these analytical results concerning the sensitivity of age-related linear aggregates to the population growth rate. It also explores the consequences of an alternative hypothesis concerning aggregation of individual productivity; based on a CES production function with imperfect substitutability between workers of different ages. It shows that sensitivities to ageing may be more important in that case, if we cumulate the fact that productivity not only differs across ages, but may also change, for each age group, as a result of the changing numerical balance between these different age groups. However, orders of magnitude remain once again rather weak.

Last, the sensitivity of average productivity to labour force ageing due to increasing the retirement age has been explored too, using once again the age profiles of figure 7. The scenario for increasing the retirement age is the same as for figures 5 and 6, and the result is given on figure 10. It is only in the case of monotonically declining productivity that the impact appears to be significantly negative: increasing the retirement age adds another loss of 4% for average productivity, once the process is fully completed. With the more realistic hump-shaped profile, increasing the retirement age is almost neutral. Productivity between 60 and 65 is lower than maximal productivity, roughly equivalent to average productivity, so that adding or removing workers of these ages to or from the labour force does not change the aggregate level of productivity.

3. Ageing and training

Do these results mean that there are no foundations to the two conventional beliefs that labour force ageing should both call for an intensification of training for older workers and significant adjustment of wage policies for these older workers? Such an inference would be, in our view, excessively rapid. We shall first examine rapidly the case for the intensification of training.

A simple model of human capital accumulation can be applied to the analysis of the optimal training policy in an ageing labour force. Assume that productivity at age a is a function f of the human capital stock $k(a)$ at this age. We shall not make any formal distinction between basic education and on-the-job training: both activities are time-consuming, $t(a)$ being the amount of time devoted to them at age a, with $t(a)$ constrained between 0 and 1. The evolution of human capital with age can therefore be described by the differential equation:

$$\dot{k}(a) = -\delta k(a) + g(t(a))$$

with the initial condition $k(0)=0$, δ being a rate of depreciation, and $g(t)$ a production function for human capital assumed to be the same at all ages (ability to learn does not decrease at later ages).

Let us assume, at last, that all time that is not devoted to training is devoted to work, until retirement age β, which is exogenously given. If $s(a)$ is the probability of surviving until age a and n the population growth rate, the optimal policy for the control variable $t(a)$ is the one which maximises output, i.e.:

$$\max_{t(a)} \int_0^{\beta} e^{-na} s(a)(1-t(a)) f(k(a))da$$

subject to the differential equation above. The solution to this optimal control program is given in appendix A2. Three simulations have been performed with the following scenarios:

- A reference scenario with $f(k)=k$, $g(t)=t^{1/2}$, $\beta=60$, $n=0$, $\delta=1\%$ and $s(a)=1$ until retirement age.
- A scenario of population decline, with the same parameters as the reference scenario except a negative population growth rate of –1%.
- A scenario of delayed retirement age: n is kept equal to zero, but β is assumed to increase from 60 to 65 years.

Results for both $t(a)$ and $k(a)$ are given on figures 11 and 12. Both forms of labour force ageing imply an increase in the optimal level of training, but for different reasons and with different modalities. Ageing due to a declining population growth rate increase training efforts during the first half of the life cycle, with an age at first entry in the labour market that is raised by about 4 years (which is not a marginal change). The direction of this change can be

Figure 11: Optimal profiles of education and on-the-job training for three demographic scenarios (Source: Blanchet, 1992)

Figure 12: Optimal stock of human capital by age for three demographic scenarios (Source: Blanchet, 1992)

explained by cost considerations. In a growing population where young age groups are numerically important, providing them with an important level of initial formation is costly and this limits the importance of this initial formation. The reason why ageing increases training efforts, therefore, is not the necessity to cope with declining productivity at later ages, but rather the fact that a declining population lowers the cost of providing young age groups with large amounts of formation: this applies to initial formation, but also to on-the-job training during the first part of active life. On the other hand, there does not appear to be any significant intensification of training efforts during the second part of active life, the reason being that the horizon on which training efforts can be valorised remains limited by the retirement age of 60.

If ageing is due to increased age at retirement, this horizon increases, and we assist to a significant increase of training intensity at later ages. On the other hand, changes in the amount of training are much more limited at younger ages.

The consistency between resulting profiles of human capital over the life cycle and the hump-shaped profile of figure 7 must be noted. In both cases, it appears that the role of additional training is not to limit some adverse consequences of ageing for average productivity. They are instead used to amplify an impact of ageing which, in this particular case, is already positive without any change in training policies. In this particular but not unrealistic case, ageing may be described as widening, rather than narrowing productivity prospects, by lowering the cost of initial formation from a collective point of view, or by making formation more rewarding, because it lengthens the time horizon over which it may be valorised. If the case for more training is preserved, its premises are strongly revised.

4. Ageing and wage policies

The other major policy issue associated to labour force ageing is the problem of wages and labour cost. The argumentation is well known. Large or small, the impact of ageing on productivity is neutral for unit labour costs or employment of older workers if, at any age, wages follow the profile of productivity. Yet there may be various reasons for a divergence between the age profiles of productivity and wages over the active life cycle. One reason for this has been proposed in the context of the human capital accumulation model provided in the last section. If part of the training effort during the first half of the active life cycle is financed or at least co-financed by the employer, then the total cost of younger workers will exceed their actual productivity, and they will have to reimburse this extra-cost at later stages of the life-cycle, by accepting wages lower than their productivity. In that case, an ageing labour force is again rather good news for employers since it should decrease the unit cost of effective labour.

But the story is the opposite if we assume that wages increase more rapidly than productivity over the life cycle. This is generally justified by agency motives, i.e. the need to establish implicit contracts between workers and their firm which maximise their (unobservable) work effort: the promise of relatively high wages at the end of the active life-cycle warrants maximum effort and loyalty to the firm by the employee, since the risk of being fired-off means the complete loss of this seniority premium. Another and simpler rationale behind such wage patterns is that workers' motivation is more efficiently sustained by wage increases than by wage levels: if total wages and total productivity are constrained to equilibrate over the

Figure 13 : Ratio productivity/wage in French manufacturing, according to age, gender and skill
(source Crepon, Deniau and Perez-Duarte, 2001)

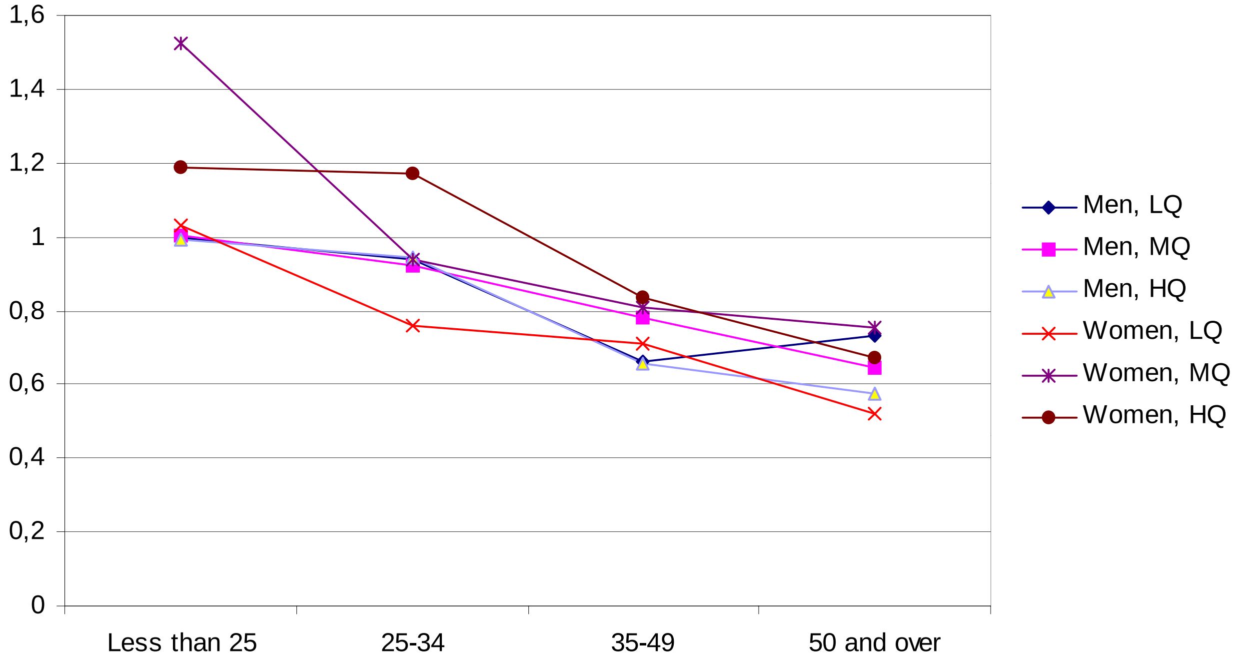
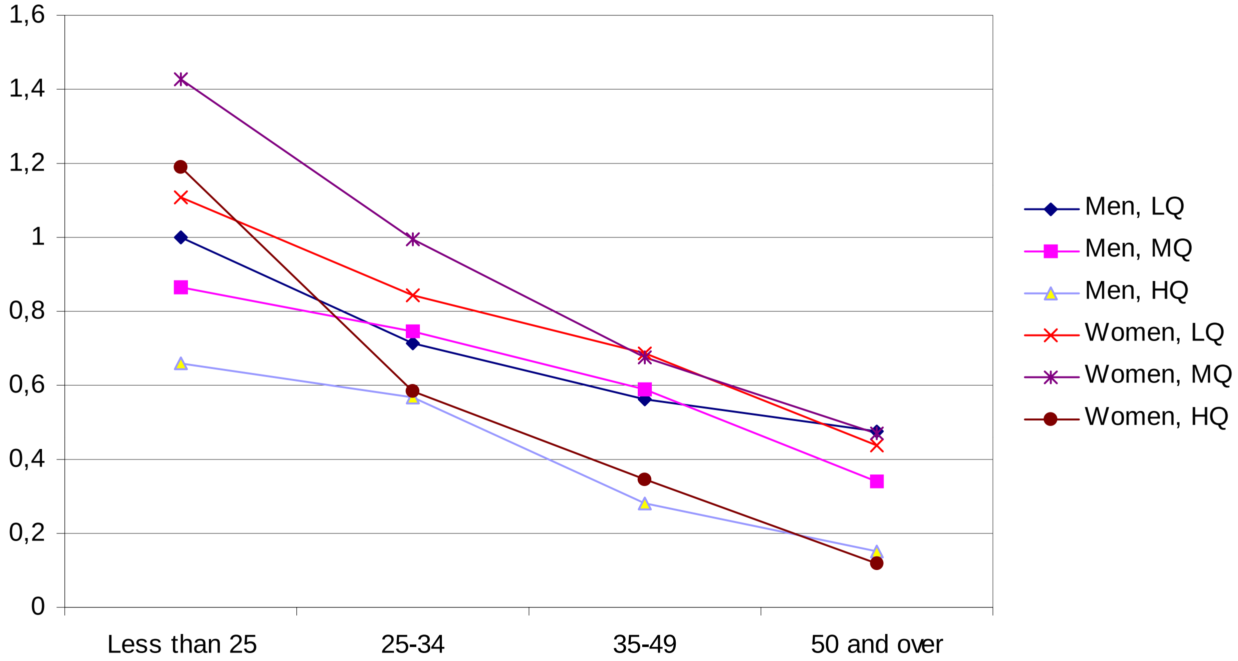


Figure 14 : Ratio productivity/wage in French services according to age, gender and skill
(source Crepon, Deniau and Perez-Duarte, 2001)



full life cycle, this necessarily imply wages below productivity at younger ages, and higher than productivity at later ages.

Although this story makes intuitive sense, it has generally been considered as difficult to validate empirically, because of lack of direct observation of productivity performance. The situation is even worse than that: it is precisely in contexts where productivity is the hardest to observe at the individual level that such contracts are the most likely to occur, and this lowers the chances to be able to validate their existence.

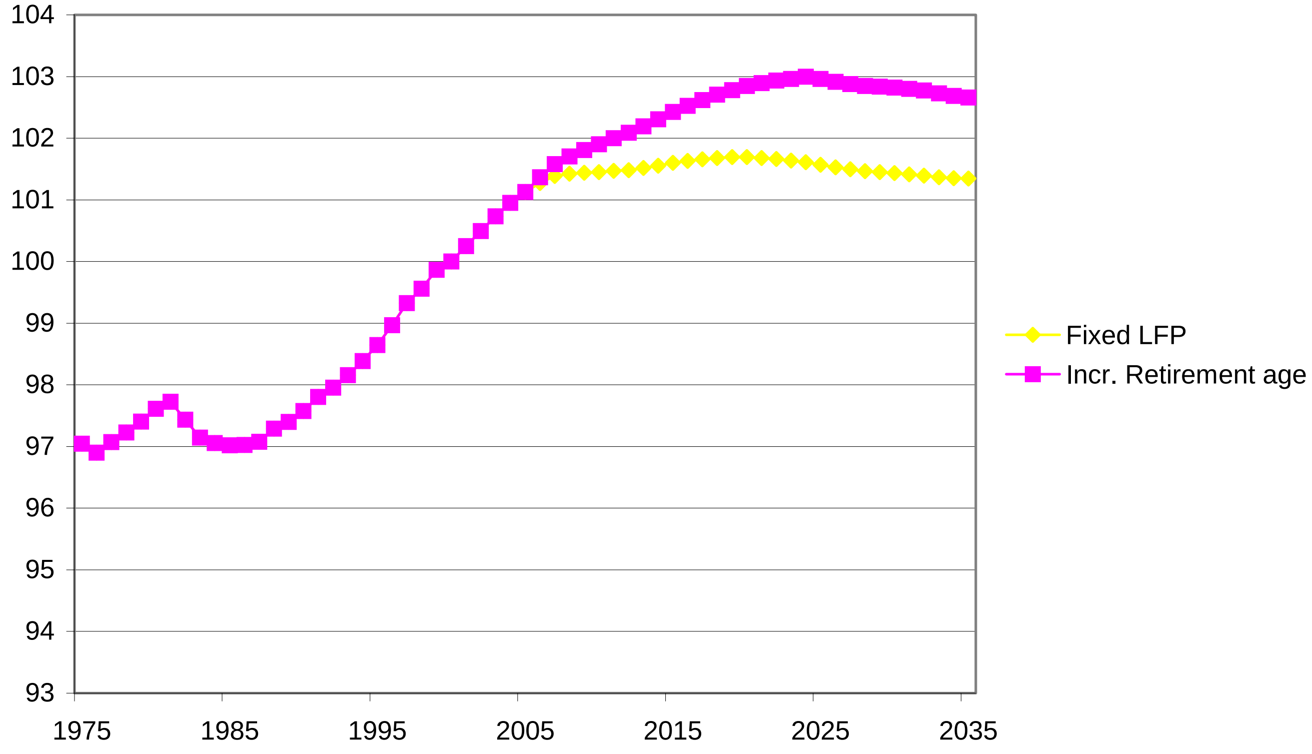
Indirect observation is therefore necessary. One recent systematic attempt is provided by the recent study by Crépon, Deniau and Perez-Duarte (2001), already quoted above, extending a methodology proposed by Hellerstein, Neumark and Troske (1999), which consist in jointly estimating wages and productivity by age and some other individual characteristics (gender and skill), from a panel of data aggregated at the firm level. Productivity and wages by individual characteristics are respectively obtained by regressing total value added and the total wage bill on the shares of firms' labour force with these various characteristics. For this purpose, age is regrouped in the four modalities already mentioned above (below 25, 25 to 34, 35 to 49, 50 and over), and qualifications in three modalities.

Results in terms of productivity profiles have been presented above, and only conclude to a slight differentiation according to age. The only exception is the strong increase of productivity between the two age groups for highly qualified workers. Wage profiles, on the other hand, and this could have been observed directly as well from individual wage data available in their data base, are very significantly increasing with age for all gender×skill groups. The authors derive from the estimates some indicators of markdown, i.e. the ratio of productivity to wages, reproduced on figures 13 and 14, which are strongly decreasing with age, being generally divided by two between the beginning and the end of the active life cycle.

For figure 15, we have used a profile of markdown very roughly derived from these estimates, and close to the one already use in Blanchet (1992), assuming a linear decline of this markdown from 1 to 0.5, and applied this assumption to a retropolation and projection of unit labour costs for the average firm, normalized to 100 in 2000, under the medium demographic scenario with fixed futures and LFP rates, and under the scenario of increasing retirement age, i.e. the same two scenarios as for figure 10.

The result of this projection may be considered, once again, as far from dramatic. The impact of ageing on average labour cost would be only to increase it by 1,5 percentage point from now on to steady state under the fixed LFP rate, and by 3 percentage points with retirement age increased by years. The fact that the order of magnitude is the same as for previous productivity projections is not a surprise. The age profile for the markdown coefficient is qualitatively and quantitatively similar to the first of the two productivity scenarios used before: the increase of unit labour costs displayed in figure 15 are therefore quantitatively comparable to the decreases in average productivity which were displayed, for this productivity scenario, on figure 10.

Figure 15 : Simulated evolution of average unit labour costs (medium demographic scenario)



The consequence is that ageing, in itself, cannot be considered as the potential source of considerable increases in labour costs, especially if we take into account the fact that the simulated increases are spread over a period of 10 or 20 years over which many other factors may affect labour costs in more significant ways. This impact of ageing on labour costs could be fully compensated by a limited amount of wage moderation.

It remains, however, that such an impact would cumulate, in particular the consequences of ageing *outside* the labour force on social contributions for pensions. Furthermore, even is the interaction between this seniority premium and age structures is weak, it also remains that these seniority premium creates a pressure for limiting the share of older workers in employment: this pressure exists even without ageing, but becomes more stringent when ageing becomes more important, or more generally when, with or without ageing, firms face a necessity to contain labour costs, as was the case since the mid 70s. French society accommodated this process, during the last decades, by intensive use of early retirement schemes. Global population ageing and its consequences on pension financing make this solution less and less desirable, calling for a new way to deal with this problem. But the nature of the difficulty must be stated clearly: it is global ageing which should put these wage profile under strain, not the share of this ageing process which occurs within the labour force, whose direct consequences remain of second order.

Appendix 1: Sensitivity of average productivity to the population growth rate: some analytical results.

1) The case of linear aggregation: general results

We consider the general case of an age-related phenomenon $x(a)$, X the associated aggregate in a stable population growing (or declining) at rate n. In this population the age structure $p(a)$ will be proportional to $s(a)\ e^{-na}$ where $s(a)$ is the probability of having survived up to age a. We have, therefore:

$$X = \int_{a=0}^{\omega} e^{-na}\ s(a)x(a)\ da \Big/ \int_{a=0}^{\omega} e^{-na}\ s(a)\ da$$

A logarithmic derivation of X with respect to n leads to:

$$\frac{\mathrm{d}(\log X)}{\mathrm{d}n} = -\frac{\int ax(a)s(a)e^{n\gamma a}\,da}{\int x(a)s(a)e^{n\gamma a}\,da} + \frac{\int as(a)e^{-na}\,da}{\int s(a)e^{-na}\,da}$$

or

$\mathrm{d}X/X = (A\text{-}A_x)\ \mathrm{d}n$

where A is the mean age of the total population (mean of the distribution $e^{-na}s(a)$) and A_x the mean age associated to the variable $x(a)$ (mean of the distribution $e^{-na}s(a)x(a)$). In the case of retirement, quoted in the text, variable $x(a)$ will be the dummy variable equal to zero before retirement age and 1 after. In this case, the gap between these two mean ages is about 30, meaning for instance that a 1 percentage point drop in the population growth rate (around n=0), will lead to a 30% increase of X, i.e. a share of retired people in the whole population multiplied by 1.3.

In the case of productivity, mean ages A_x associated to the two productivity profiles used in the text are respectively equal to 37.0 and 40.8 years. The global mean age A to be used is the mean age of the labour force, rather than the global mean age, which can be found on figure 4: it was 39.4 years in 2000[4]. The result is a difference A-A_x equal to either +2.4 or -1.4 years, implying that a *drop* of 1% of the population growth rate implies either 2.4% less or 1.4 more in terms of productivity. These orders of magnitude are consistent with results of figures 8 and 9 if we remind that a 1% drop of n roughly corresponds to a drop of fertility from 2.1 to 1.5.

[4] These mean ages are those obtained for the current age structures, taken as approximations of those that would be found in stable age structures.

2) Aggregation of age-specific productivities with a CES production function

We still assume a stable population growing (or declining) at rate n. We now neglect mortality during active life and consider for simplicity full activity between extreme ages of labour force participation. The number of workers $p(a)$ is therefore equal to e^{-na} and, instead of a linear production function, we now consider the CES function:

$$Y = \left[\int \pi(a)\, p(a)^{-\gamma}\, da\right]^{-1/\gamma} = \left[\int \pi(a) e^{n\gamma a} da\right]^{-1/\gamma}$$

where $\pi(a)$ is the age-specific productivity profile and where $\sigma=1/(1+\gamma)$ measures the substitutability between the different age groups. $\gamma=-1$ implies $\sigma=\infty$ and corresponds to a perfect substitutability between workers of all ages (which does not exclude some productivity differentials measured by $\pi(a)$): this is the situation considered in the text. $\gamma=0$ implies $\sigma=0$ and correspond to a Cobb-Douglas production function. $\gamma\rightarrow+\infty$ correspond to the extreme (and obviously unrealistic) case of perfect complementarity between age groups ($\sigma=0$). This specification is not really satisfactory: it assumes that the elasticity of substitution between workers of two different age groups is independent of the age gap between these two groups, an assumption which is clearly unacceptable. This specification, therefore, does not intend to do more than providing an exploration of differences generated when relaxing the assumption of linearity.

Under this assumption, average productivity is equal to:

$$y = \left[\int \pi(a) e^{n\gamma a} da\right]^{-1/\gamma} \Big/ \left[\int e^{-na} da\right]$$

From this equation, we can again compute a logarithmic derivative with respect to n. We get:

$$\frac{\mathrm{d}(\log y)}{\mathrm{d}n} = -\frac{\int a\pi(a) e^{n\gamma a} da}{\int \pi(a) e^{n\gamma a} da} + \frac{\int a e^{-na} da}{\int e^{-na} da}$$

The second term of this derivative is once again the mean age of the labour force, the first one can be interpreted as the mean age associated to the distribution $\pi(a)\mathrm{e}^{n\gamma a}$. We observe that, for $\gamma=-1$, we retrieve the expression directly obtained with the linear specification.

We can first consider the case where $\pi(a)=1$ for all a. In that case, if $\gamma=-1$, the derivative is identically equal to zero so that the age structure of the population is completely neutral for average productivity. For $\gamma>-1$, it is easy to see from figures A.1.a and b that the profile of the function $e^{n\gamma a}$ will systematically give more weight (respectively less) to higher ages for $n>0$ (respectively $n<0$) so that the derivative of y with respect to n will be negative if n is positive and positive if n is negative, implying, as expected, that a uniform age distribution is optimal ($n=0$).

Now, if π(a) is no more uniform across ages, the optimality of n=0 does not hold anymore. We can consider the particular case of exponential profiles for $\pi(a)$, i.e. $\pi(a)=e^{\delta a}$. Table A.1 shows values of d(log y)/dn around various values of n and for various values of γ and δ. An

Figure A.1: Profiles for e^{-na} and $e^{n\gamma a}$, depending on values of n and γ.

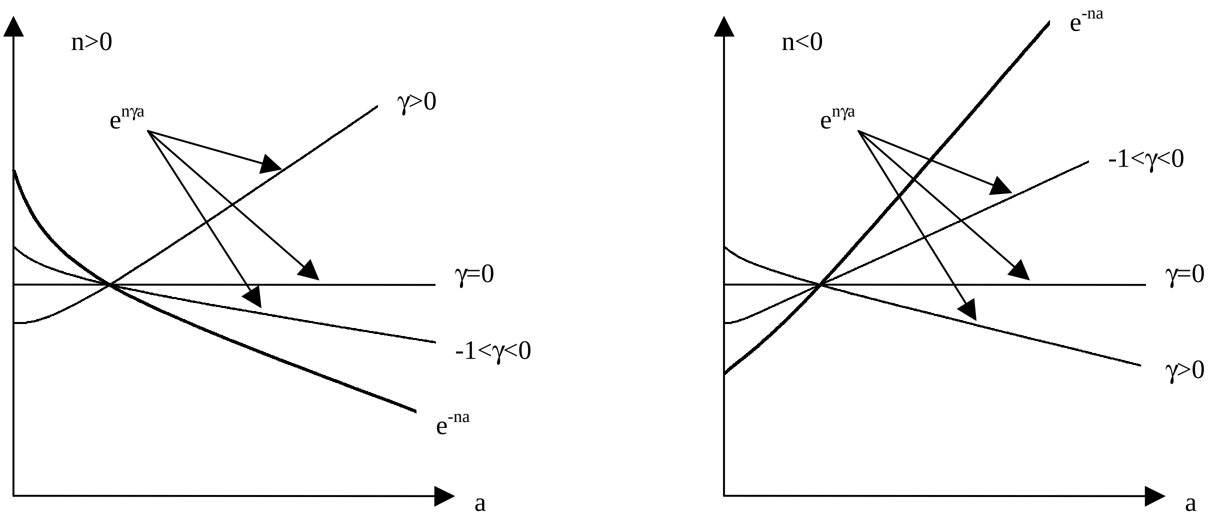


Table A.1: Logarithmic derivative of y with respect to n, for various values of n, γ and δ.

		n		
γ	δ	-0.01	0.00	0.01
	-0.01	1.40	1.40	1.37
-1.0	0.00	0.00	0.00	0.00
	0.01	-1.37	-1.40	-1.40
	-0.01	2.10	1.40	0.69
-0.5	0.00	0.70	0.00	-0.70
	0.01	-0.69	-1.40	-2.10
	-0.01	2.79	1.40	0.00
0	0.00	1.40	0.00	-1.40
	0.01	0.00	-1.40	-2.79
	-0.01	3.48	1.40	-0.70
0.5	0.00	2.10	0.00	-2.10
	0.01	0.70	-1.40	-3.48
	-0.01	4.17	1.40	-1.40
1.0	0.00	2.79	0.00	-2.79
	0.01	1.40	-1.40	-4.17

increasing δ (more productivity at later ages) shifts the optimal n to the left: an older age structure becomes preferable. But the most important result of this table is to provide extreme bounds for the sensitivity of y to n. If we admit that the table covers the range of admissible values for the different parameters, we find that the logarithmic derivative is bounded by a little more than 4 in absolute value. This means that permanent changes of n of 1 percentage point, around values of n comprised between –1% and +1%, lead to changes of y which are lower than 4% in absolute value, i.e. twice as much as order of magnitudes obtained in the linear case: this remains low.

Appendix 2: Optimal training

We must solve the following program:

$$\max_{t(a)} \int_0^{\beta} e^{-na} s(a)(1-t(a)) f(k(a)) da$$

Subject to:

$$\dot{k}(a) = -\delta k(a) + g(t(a))$$
$$k(0) = 0$$
$$0 \leq t(a) \leq 1$$

Introducing the co-state variable $p(a)$ we get the Hamiltonian:

$$H(k,t,p) = e^{-na} s(a)(1-t(a)) f(k(a)) da + p(a)[g(t(a)) - \delta k(a)]$$

The optimal solutions for $p(a)$ and $t(a)$ must satisfy the two conditions:

$$\dot{p}(a) = -\frac{\partial H}{\partial k} = -e^{-na} s(a)(1-t(a)) f'(k(a)) + \delta p(a)$$

and (maximisation of *H* with respect to $t(a)$):

$$t(a) = 0 \qquad \text{if } -s(a) f(k(a)) e^{-na} + p(a) g'(0) < 0$$
$$t(a) = 1 \qquad \text{if } -s(a) f(k(a)) e^{-na} + p(a) g'(1) > 0$$
$$t(a) = g'^{-1}\left(\frac{s(a) f(k(a)) e^{-na}}{p(a)} \right) \qquad \text{otherwise}$$

plus the transversality condition $p(\beta)=1$.

For practical applications, we have taken the following functional forms: $g(t)=t^{0,5}$ and $f(k)=k$. Mortality before retirement age has been neglected, i.e. $s(a)=1$ for all *a*.

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